Cognitive Science 28 (2004) 451–466

COGNITIVE SCIENCE

http://www.elsevier.com/locate/cogsci

You'll see what you mean: Students encode equations based on their knowledge of arithmetic

Nicole M. McNeil*, Martha W. Alibali

Department of Psychology, University of Wisconsin-Madison, 1202 West Johnson Street, Madison, WI 53706, USA

Received 13 May 2003; received in revised form 31 October 2003; accepted 4 November 2003

Available online 5 March 2004

Abstract

This study investigated the roles of problem structure and strategy use in problem encoding. Fourth-grade students solved and explained a set of typical addition problems (e.g., $5 + 4 + 9 + 5 =$ __) and mathematical equivalence problems (e.g., $4 + 3 + 6 = 4 +$ __ or $6 + 4 + 5 =$ __ $+ 5$). Next, they completed an encoding task in which they reconstructed addition and equivalence problems after viewing each for 5 s. Equivalence problems of the form $4 + 3 + 6 = 4 +$ __ overlap with a perceptual pattern found in traditional arithmetic problems (i.e., answer blank in final position), and students' encoding was poorest on problems of this type. Individual differences in encoding the equivalence problems were related to variations in strategy use. Some students solved blank-final equivalence problems using the standard arithmetic strategy of performing all given operations on all given numbers. These students made more errors in encoding problem structure, but fewer errors in encoding the numbers, than did students who solved the problems using correct or other incorrect strategies. Moreover, students who expressed many strategies for solving the blank-final equivalence problems made fewer errors in encoding problem structure, but more errors in encoding the numbers, than did students who expressed only a single strategy. Results highlight that encoding is intended to guide action and that prior experience can simultaneously facilitate and interfere with accurate encoding.
© 2003 Cognitive Science Society, Inc. All rights reserved.

Keywords: Encoding; Problem solving; Knowledge acquisition; Mathematics education

1. Introduction

To solve a problem correctly, a solver must encode, or internally represent, the important features of the problem (Chase & Simon, 1973; Kaplan & Simon, 1990; Siegler, 1976).

* Corresponding author. Tel.: +1-608-263-0757; fax: +1-608-262-4029.
E-mail addresses: nmmcneil@wisc.edu (N.M. McNeil), mwalibali@wisc.edu (M.W. Alibali).

0364-0213/$ – see front matter © 2003 Cognitive Science Society, Inc. All rights reserved.
doi:10.1016/j.cogsci.2003.11.002

However, solvers do not always encode all problem features accurately. Indeed, it is not uncommon for different individuals, given the same problem, to encode different features. Research suggests that individual differences in encoding depend on at least two factors: (1) the extent to which a given problem conforms to the perceptual patterns individuals have encountered in past problem-solving experiences (Chase & Simon, 1973; Knoblich, Ohlsson, Haider, & Rhenius, 1999) and (2) the strategies individuals use to solve the problems (Siegler, 1976). Prior studies regarding the effects of problem structure and strategy use on encoding have focused exclusively on cases in which prior problem-solving experience facilitates future success. The present paper examines how problem structure and strategy use are related to encoding in a domain in which prior experience does not always lead to success on future problems—the domain of mathematics.

1.1. Encoding and the match between problem structure and previously-encountered perceptual patterns

Mathematics is a domain in which early competence with "basic skills" (e.g., arithmetic operations) is thought to be necessary for advanced thinking and problem solving. Traditional American mathematics curricula are designed around this assumption. Students learn arithmetic operations for many years before they are introduced to complex equations. Indeed, data from the Third International Mathematics and Science Study (Beaton et al., 1996) indicate that American students spend substantial class time reviewing basic arithmetic skills throughout the elementary and middle school years (see also Valverde & Schmidt, 1997). Given this early and elongated experience with arithmetic, students are well versed in the perceptual patterns of arithmetic problems by late elementary school.

In some respects, the perceptual patterns encountered in arithmetic problems are not predictive of those encountered in more complex equations. In arithmetic problems, the equal sign and answer blank are regularly presented at the end of the problem (e.g., $4 + 7 = \_$; $4 - 3 + 6 = \_$). We propose that students extract this perceptual pattern from their experience with arithmetic problems. Past research suggests that this should lead to fast and accurate encoding when problems conform to the familiar pattern (e.g., Chase & Simon, 1973). However, many higher-level problems do not conform to the pattern. For example, in the problem "$2 \times \_ + 6 = 20$" neither the equal sign nor the answer blank is at the end of the problem. Thus, knowledge of the perceptual pattern cannot help (and may hurt) students' performance.

When knowledge derived from early experience does not correspond to information encountered in future endeavors, the likelihood of error increases. Traditionally, this is referred to as the effect of "top-down" processing (e.g., Bruner, 1957; Rumelhart, 1980). Contemporary theorists (e.g., Kuhl, 2000; Zevin & Seidenberg, 2002) explain this effect in terms of the "entrenchment" of learned patterns. According to this view, the patterns with which individuals gain experience become entrenched, and subsequently, novel information that overlaps with the patterns is assimilated to those patterns. This is beneficial when information matches the learned patterns, but can lead to errors when it does not (Best, McRoberts, & Goodell, 2001; Flege, Yeni Komshian, & Liu, 1999). If a stimulus overlaps with an entrenched pattern but does not match it exactly, the stimulus may be encoded inaccurately. However, if a stimulus bears little resemblance to an entrenched pattern, an individual may rely on sensory information (i.e., "bottom-up" processes) to encode the stimulus features. Thus, the match between entrenched perceptual patterns and the structure of external stimuli determines how information is encoded.

In the present study, we focus on the perceptual pattern of having "$=$ __" at the end of the problem. We predict that the match between this pattern and a given problem's structure will influence encoding. If a problem matches the pattern, it will be encoded accurately. If a problem overlaps with the pattern but does not match it exactly, it will be assimilated and encoded inaccurately. If a problem neither matches nor overlaps with the pattern, it will not be assimilated, and encoding errors will be less likely and less systematic.

We focus on fourth-grade students' encoding of two classes of problems: (1) typical addition problems (e.g., $3 + 4 + 5 + 3 =$ __), which match the pattern of having the equal sign and answer blank at the end of the problem, and (2) mathematical equivalence problems, which have addends on both sides of the equal sign, and therefore, do not match this pattern. We used two types of equivalence problems: those with the blank in final position (e.g., $3 + 4 + 5 = 3 +$ __) and those without the blank in final position (e.g., $3 + 4 + 5 =$ __ $+ 5$). We chose fourth-grade students because they have had years of experience with arithmetic, but no experience with higher-level problems such as algebraic equations. Mathematical equivalence problems are not typically included in traditional elementary mathematics curricula (Perry, 1988).

Let us consider how equivalence problems relate to the pattern of having "$=$ __" at the end. Blank-final problems overlap with the pattern, whereas non-blank-final problems do not. If the perceptual pattern becomes entrenched as a result of students' early experience with arithmetic, fourth-grade students should assimilate the blank-final problems to the pattern and misencode them (as if they are typical addition problems). Students should be less likely to misencode the non-blank-final problems because they do not overlap with the pattern (cf. Best et al., 2001).

The extent to which a given problem conforms to previously encountered perceptual patterns is not the only source of systematic differences in problem encoding. Indeed, there are surely individual differences in encoding even within a given problem type. Past work suggests that strategy use is highly correlated with encoding performance. We turn next to this issue.

1.2. Encoding and strategy use

Many theories predict systematic relations between encoding and strategy use. Previous studies of this issue have examined performance as a function of age or expertise (Chi, 1978; Chi, Feltovich, & Glaser, 1981; Dean & Malik, 1986; Morales, Shute, & Pellegrino, 1985; Siegler & Chen, 1998). Older children and experts tend to use more sophisticated strategies, and to encode problems better, than younger children and novices.

There are several potential interpretations of the observed relations between encoding and strategy use, including the possibility that they are due to a third factor, such as time on task. However, in this paper, we focus on two specific accounts that posit direct links between encoding and strategy use. The first is based on the skill acquisition literature, and we refer to it here as the "freed-resources account." This account assumes a fixed-capacity model of attention, and suggests that the more highly practiced and proceduralized a strategy is, the fewer cognitive resources are needed for implementing it (Kotovsky, Hayes, & Simon, 1985; Shiffrin & Schneider, 1977). According to this view, when a strategy is highly practiced, resources are "freed up," allowing the solver to encode other aspects of the stimulus, including important problem features and relations among those features (cf. Shrager & Siegler, 1998). The second account is based on the assumption that encoding is intended to guide action. We refer to it as the "action-primacy account." This account suggests that when a strategy is highly practiced, the solver will encode only those problem features necessary for executing the strategy (cf. Fowler, 1986; Gibson, 1979).

Prior studies cannot distinguish between these accounts because they have focused exclusively on domains in which correct strategies are well learned or highly practiced. If an individual with a highly-practiced correct strategy encodes problems more accurately than an individual with an incorrect or inefficient strategy, it could be due either to the individual having sufficient resources available to encode all of the problem features, or to the individual encoding the features needed to execute the correct strategy. The present study distinguishes between these alternatives because it focuses on a domain in which the most highly practiced strategy is an incorrect one. If encoding depends on resource availability, then students who use a highly-practiced strategy should encode problems accurately. In contrast, if encoding depends on the strategies students use, then students who use a highly-practiced strategy should encode problems poorly.

Through their experience with typical arithmetic problems, students learn to perform all given operations on all given numbers to reach a solution (e.g., $2 \times 3 + 7 = 13$). After years of practice, this strategy is highly proceduralized. However, performing all given operations on all given numbers is not an appropriate strategy for solving certain equations, such as "$3 + 4 + 5 = 3 + \_$." Nevertheless, when solving such a problem, many elementary students add all the numbers and put "15" in the blank (Alibali, 1999; McNeil & Alibali, 2000, 2002).

Students' use of this "add all" strategy provides us with a unique opportunity to examine whether the freed-resources account or the action-primacy account better characterizes encoding performance. The freed-resources account predicts that students who use the highly-practiced "add all" strategy should encode problems accurately. In contrast, the action-primacy account argues that solvers should encode only those problem features necessary to implement their solution strategy. For the "add all" strategy, the only necessary features are the operators (e.g., plus signs) and the numbers. Thus, students who use the "add all" strategy should encode the operators and numbers accurately, but they may encode aspects of the problem structure, such as the position of the equal sign, inaccurately (e.g., they may misencode $3 + 4 + 5 = 3 + \_$ as $3 + 4 + 5 + 3 = \_$, but not as $3 \times 4 \times 6 = 3 \times \_$).

In examining the relationship between encoding and strategy use, we consider the entire repertoire of strategies that students display across problems (Alibali, 1999; Alibali, McNeil, & Perrott, 1998). This allows us to examine relations between strategy variability and encoding. The freed-resources account predicts that encoding performance should decrease as strategy variability increases. Individuals expend resources in managing multiple, competing strategies and in selecting one to execute (Goldin-Meadow, Alibali, & Church, 1993; Goldin-Meadow, Nusbaum, Garber, & Church, 1993; Siegler, 1989), so as the number of strategies in their repertoires increases, they have fewer resources available for encoding. Conversely, the action-primacy account predicts that encoding performance should increase as strategy variability increases. According to this account, individuals allocate attention to the problem features necessary for executing potential strategies (Luchins & Luchins, 1959). Thus, as the number of strategies in individuals' repertoires increases, the number of problem features they encode also increases.

In sum, the present study examines the relationship between problem encoding and (1) the extent to which given problems match the perceptual patterns encountered in past problem-solving experiences and (2) the strategies used to solve the given problems. In keeping with past research (e.g., McNeil & Alibali, 2000; Siegler, 1976), we assess encoding and strategy use with two separate tasks: problem reconstruction and problem solution.

2. Method

2.1. Participants

Participants were 70 fourth-grade students (28 boys, 42 girls; age $M = 10;2$, range 9;6–10;10). Students were drawn from four urban parochial schools that used traditional mathematics curricula.

2.2. Stimuli

Stimuli were typical addition problems and mathematical equivalence problems arranged in mixed blocks. Each block consisted of one problem of each of the four types shown in Table 1. Problems were not repeated across blocks.

2.3. Procedure

Students were tested individually. They completed two tasks in fixed order: (1) a problem-solving test (used to assess strategy use) and (2) a reconstruction test (used to assess encoding).[1] In the problem-solving test, 12 problems (three blocks, each consisting of one problem of each of the four types) were presented one at a time by an experimenter at the blackboard. Students solved each problem and explained their solutions. In the reconstruction test, 16 novel problems (four blocks, each consisting of one problem of each of the four types) were projected onto a plain wall for 5 s each. After each problem was removed, students wrote the problem on the blackboard. Students were instructed to write each problem "exactly as they saw it." Students received no feedback about accuracy on either task. After the session, each student received a brief lesson about mathematical equivalence.

Table 1
Problems used in the study

Problem type	Example
Typical addition with repeated addend	$3 + 6 + 7 + 3 = \_\_$
Typical addition without repeated addend	$4 + 8 + 5 + 7 = \_\_$
Blank-final equivalence	$4 + 3 + 5 = 4 + \_\_$
Non-blank-final equivalence	$6 + 8 + 4 = \_\_ + 4$

Table 2
Incorrect and correct strategies for solving the problem $3 + 4 + 5 = \_\_ + 5$

Strategy	Solution	Sample verbal explanation	Sample gestured explanation
Add all	17	"I added 3 and 4 and 5 and 5."	Right-hand point to 3, 4, left 5, right 5.
Add to equal sign	12	"I added 3 plus 4 plus 5, and that equals 12."	Right-hand point to 3, 4, left 5, solution.
Add to equal sign & add all	12 & 17	"I added 3 and 4 and 5, and I put 12 in the blank, and then I added the 5 and put down 17."	Right-hand point to 3, 4, left 5, first solution, right 5, second solution.
Carry	3	"There was a 3 here, so I put 3."	Right-hand point to 3, solution.
Idiosyncratic	1	"I just guessed."	No gesture.
Equalize	7	"3 plus 4 plus 5 equals 12, and 7 plus 5 equals 12."	Right-hand point to 3, 4, left 5, hand down. Left-hand point to solution, right 5.
Add–subtract	7	"I added 3 plus 4 plus 5, and then I took 5 away from that."	Right-hand point to 3, 4, left 5, hand down. Left-hand point to right 5.
Group	7	"I saw the two 5s, so I just added the 4 and the 3, and that's 7."	Right-hand point to right 5 and left-hand point to left 5, hands down. Right-hand point to 4, 3, solution.

2.4. Coding

2.4.1. Strategy use

For each equivalence problem, we identified the strategy each student used to arrive at a solution. For many problems, this could be inferred from the solution itself (e.g., for $3 + 4 + 5 = 3 + \_\_$, if the student's solution was 15, the "add all" strategy was inferred). If the solution was ambiguous, the strategy was assigned based on the explanation provided in speech, and if both solution and speech were ambiguous, the strategy was assigned based on the explanation provided in gesture. We coded spoken and gestured problem explanations using the system developed by Perry, Church, and Goldin-Meadow (1988). Strategies fell into eight categories, shown in Table 2.

2.4.2. Primary strategy groups

Students were categorized into one of three primary strategy groups based on the strategies they used to solve the equivalence problems. Many students ($N = 30$) used different strategies for blank-final and non-blank-final problems, so primary strategy groups were assigned separately for the two problem types. Students in the *correct* group (blank-final $N = 29$; non-blank-final $N = 24$) used a correct strategy (e.g., "equalize") consistently on at least two of the three problems of a single type. Students in the *add-all* group (blank-final $N = 29$, non-blank-final $N = 11$) used the "add all" strategy on at least two of the three problems of a single type. Students in the *other-incorrect* group (blank-final $N = 11$, non-blank-final $N = 35$) used one of the other incorrect strategies (e.g., "carry") consistently on at least two of the three problems of a single type. One student was excluded from the analyses involving primary strategy because she did not use one strategy consistently on at least two of the three problems of either type.

2.4.3. Strategy repertoire

We defined the strategy repertoire as the complete set of strategies that a student expressed when explaining his or her solutions to problems of a single type. As in past work (Alibali, 1999; Alibali et al., 1998), we considered students' spoken and gestured explanations as well as their problem solutions in evaluating strategy repertoires. We assessed repertoires separately for the two types of equivalence problems. Students were categorized based on the total number of different strategies they expressed: *one strategy* (blank-final $N = 24$; non-blank-final $N = 27$), *two different strategies* (blank-final $N = 32$; non-blank-final $N = 33$), or *three different strategies* (blank-final $N = 13$; non-blank-final $N = 9$). One student was excluded from the analyses involving strategy repertoires because she expressed more than three different strategies for problems of a single type. It was the same student who was excluded from the analyses involving primary strategy (see above).

2.4.4. Encoding

Each reconstruction was scored as correct or incorrect, and errors were classified as either *number* errors or *conceptual* errors. As shown in Table 3, number errors involved misencoding the numbers or the order of the numbers in the problem. Conceptual errors involved misencoding the structure of the equation (e.g., omitting the equal sign). Cases in which the right addend was omitted altogether and cases in which equivalence problems were converted to typical addition problems were classified as conceptual errors.

2.4.5. Reliability

To establish reliability, a second coder rescored the data for 17 participants. For the reconstruction test, reliability was 98% ($N = 272$) for classifying reconstructions as correct or incorrect and 99% ($N = 88$) for classifying errors as number or conceptual errors. For the problem-solving test, reliability was 93% ($N = 102$) for strategies expressed in speech on equivalence problems. For strategies expressed in gesture, the initial reliability sample revealed marginal reliability (73%), so the entire dataset was re-coded by a second coder, and all discrepancies between the coders were examined and resolved by a third coder. These checked gesture codes were used in evaluating students' strategy repertoires. Reliability was 100% ($N = 34$) for primary strategy group and 94% ($N = 34$) for strategy repertoire group.

Table 3
Reconstruction errors

Error type	Example
Number errors	$4 + 3 + 6 = 4 + \_\_$
	$4 + 5 + 3 = 4 + \_\_$
Conceptual errors	$4 + 3 + 5 + 4 = \_\_$
	$4 + 3 + 5 + 4 \_\_$
	$4 + 3 + 5 = + \_\_ 4$
	$4 + 3 + 5 = 4 \_\_$
	$4 + 3 + 5 = \_\_$

Note. All examples are for the given problem $4 + 3 + 5 = 4 + \_\_$.

3. Results

Unless otherwise noted, all reported statistics are significant with alpha set at .05.

3.1. Overall patterns

3.1.1. Problem solving

On the typical addition problems, all students used the correct strategy of adding up all the numbers. On the six equivalence problems, there was a bimodal distribution, with most students solving at least five problems either correctly ($N = 24$) or incorrectly ($N = 39$). Seven students solved between two and four equivalence problems correctly. Four of these seven were unsystematic in their use of a correct strategy, and the remaining three switched from an incorrect to a correct strategy across blocks. These three students who improved across blocks were excluded from the analyses involving primary strategy and strategy repertoire; however, results are unchanged if they are not excluded. One of these three students was the student who had already been excluded for reasons mentioned in the coding section; thus, the final *N* for these analyses was 67.

3.1.2. Encoding

Encoding performance was highly variable across students. Collapsing across the 16 problems, students made more conceptual errors ($M = 4.97$, $SE = 0.52$) than number errors ($M = 1.50$, $SE = 0.17$). Students' total number of *conceptual* errors (i.e., errors in encoding the structure of the equation) spanned the range from 0 to 16 (median = 4), with each value represented by at least one student. On typical addition problems, the most common conceptual error was omitting the equal sign (e.g., reconstructing $3 + 4 + 5 + 3 = \_\_$ as $3 + 4 + 5 + 3$). On equivalence problems, students made a wide variety of conceptual errors. They sometimes assimilated the problems to the perceptual pattern of having "$= \_\_$" at end and converted equivalence problems to typical addition problems. Students also made less systematic conceptual errors, such as omitting the equal sign, omitting the right plus sign, or omitting the right addend (e.g., reconstructing $3 + 4 + 5 = 3 + \_\_$ as $3 + 4 + 5 = 3\_\_$). Students' total number of *number* errors (i.e., errors in encoding the particular numbers in the equation) ranged from 0 to 6 (median = 1), with each value within that range represented by at least one student.

Students' encoding improved across blocks, $F(3, 207) = 13.28$, $MSE = 0.55$. The improvement was as would be expected with gradual learning over time, and there was no evidence of an abrupt shift in students' reconstruction errors across any of the blocks (first block $M = 1.71$, $SE = 0.16$; second block $M = 1.27$, $SE = 0.16$, third block $M = 1.13$, $SE = 0.14$, fourth block $M = 0.96$, $SE = 0.14$).

3.2. Effects of problem type on encoding

We conducted a 4 (problem type: blank-final equivalence, non-blank-final equivalence, typical addition-repeated addend, and typical addition-non-repeated addend) × 2 (error type: conceptual or number) repeated measures ANOVA, with number of encoding errors as the dependent measure. There were significant main effects of problem type, $F(3, 207) = 56.81$,

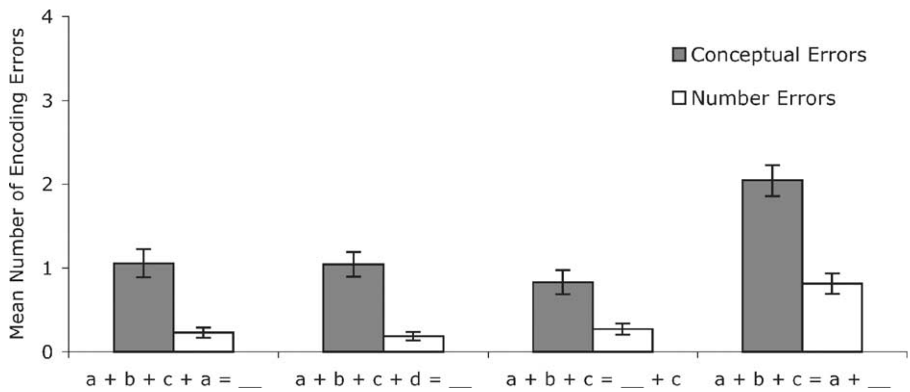

Fig. 1. Mean number of conceptual and number encoding errors students produced on each of the four problem types. The error bars represent standard errors.

MSE $= 0.42$, and error type, $F(1, 69) = 33.16$, *MSE* $= 3.18$, as well as their interaction, $F(3, 207) = 3.02$, *MSE* $= 0.88$.

As predicted, students' encoding varied systematically across problem types (see Fig. 1). Students made more errors encoding blank-final equivalence problems than non-blank-final equivalence problems or typical addition problems. For blank-final problems, students sometimes reconstructed the problems as typical addition problems ($M = 0.80$, $SE = 0.13$). This type of error was exceedingly rare on non-blank-final problems ($M = 0.03$, $SE = 0.02$). Of the 52 students who made at least one conceptual error on the blank-final problems, 30 (58%) reconstructed at least one as a typical addition problem. Of the 32 students who made at least one conceptual error on non-blank-final problems, only one (3%) reconstructed at least one as a typical addition problem.

Overall, students made more conceptual errors than number errors. However, the strength of this effect varied across problem types. As seen in Fig. 1, the greatest difference between conceptual and number errors was on blank-final equivalence problems, and the smallest difference was on non-blank-final equivalence problems. This may be due to students' accurate encoding performance on non-blank-final problems (i.e., floor effects on errors). In the following section, we consider performance for each equivalence problem type separately, and we examine the relationship between encoding and strategy use.

3.3. Relations between encoding and strategy use

Prior research has shown that students who use correct strategies to solve problems encode the problems more accurately than students who use incorrect strategies. This finding was replicated in the present study. For present purposes, however, the crucial comparison is between students who used the highly-practiced "add all" strategy and those who used other incorrect strategies. According to the freed-resources account, students who used "add all" should encode problems more accurately than students who used other incorrect strategies. In contrast, according to the action-primacy account, students who used "add all" should encode the numbers, but not the problem structure, more accurately than students who used other incorrect strategies.

We also examined the effects of strategy variability on encoding. According to the freed-resources account, if a student's repertoire contains multiple strategies, the student will expend resources in managing the competing strategies and selecting one to execute. He or she may not have sufficient resources available for encoding extraneous problem features, so encoding should be relatively poor. In contrast, according to the action-primacy account, such a student will consider multiple strategies, and this will require attention to the problem features needed for each potential strategy. Consequently, encoding should be relatively good.

3.3.1. Blank-final problems

To examine relations between primary strategy group, strategy variability, and encoding performance, we conducted a 3 (primary strategy group: correct, add-all, or other-incorrect) $\times$ 3 (strategy repertoire group: one, two, or three) $\times$ 2 (error type: conceptual or number) mixed-factor ANOVA ($N = 67$), with repeated measures on error type, and number of encoding errors on the blank-final equivalence problems as the dependent measure. The effect of error type reported above was significant, $F(1, 58) = 11.13$, $MSE = 2.00$. Three additional effects were significant: the main effect of primary strategy group, $F(2, 58) = 4.72$, $MSE = 0.46$, the interaction of error type and primary strategy group, $F(2, 58) = 3.29$, $MSE = 2.00$, and the interaction of error type and strategy repertoire group, $F(2, 58) = 5.23$, $MSE = 2.00$.

As predicted, students' primary strategy group was associated with encoding performance (see Fig. 2). A planned contrast indicated that students in the correct group made fewer errors overall than students in the two incorrect groups, $F(1, 58) = 18.23$, replicating prior research. However, the comparison of greatest interest is between students who used "add all" and students who used other incorrect strategies. According to the action-primacy account, students who used "add all" should encode the numbers accurately, but the problem structure inaccurately. The interaction of error type and primary strategy group supports this prediction. Moreover, the planned interaction contrast examining the difference in errors between students in the add-all group and students in the other-incorrect group indicates that, as expected,

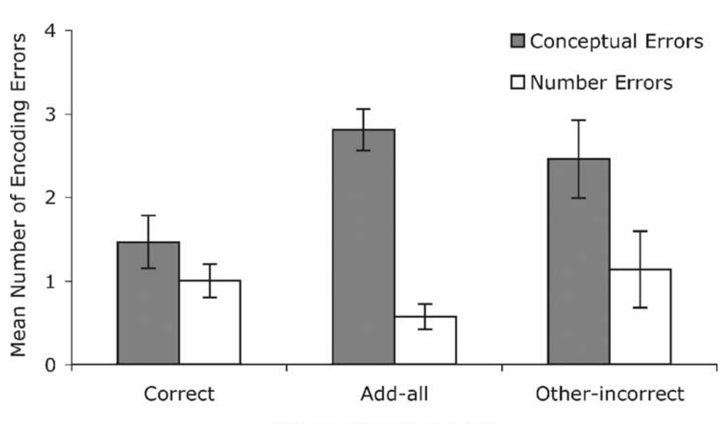

Fig. 2. Mean number of conceptual and number encoding errors as a function of the primary strategy used to solve the blank-final problems on the problem-solving test (correct, add all, or other incorrect). The error bars represent standard errors.

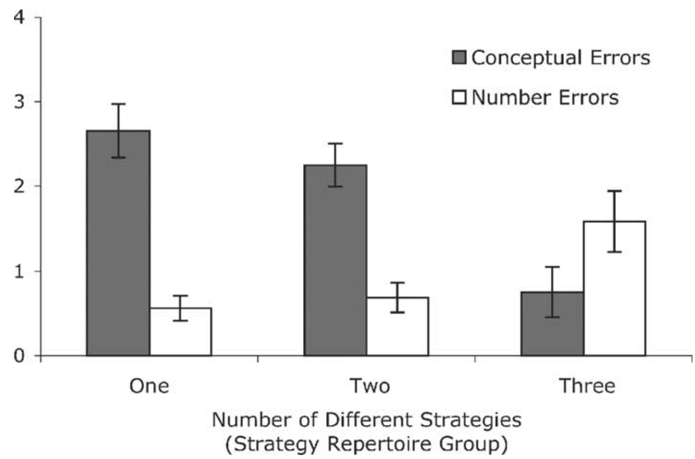

Fig. 3. Mean number of conceptual and number encoding errors as a function of strategy repertoire group on blank-final problems on the problem-solving test. The error bars represent standard errors.

students in the add-all group made more conceptual errors but fewer number errors than students in the other-incorrect group, $F(1, 58) = 7.37$, $MSE = 2.00$.

Students in the add-all group frequently encoded equivalence problems as typical addition problems. Considering students who made at least one conceptual error on the blank-final problems, 19 of 25 students in the add-all group (76%) reconstructed at least one blank-final problem as a typical addition problem (e.g., reconstructed "$3 + 4 + 8 + 3 =$ __" as "$3 + 4 + 8 = 3 +$ __"), whereas only 4 of 10 students (40%) in the other-incorrect group did so, $\chi^2(1, N = 35) = 4.11$. Thus, when they made errors, students in the add-all group often encoded only those features of equivalence problems needed to execute their highly-practiced strategy.

Strategy variability was also associated with encoding performance; however, the relationship depended on error type (see Fig. 3). Conceptual errors tended to decrease, but number errors tended to increase, as number of strategies increased. Thus, the size of students' strategy repertoire was both positively and negatively related to encoding. We return to this issue in the discussion.

3.3.2. Non-blank-final problems

For the non-blank-final problems, a parametric analysis would be unwarranted because of floor effects (i.e., few encoding errors), so we used non-parametric statistics to examine the relationship between strategy use and encoding. We used logistic regression to predict the probability that students made at least one conceptual encoding error. The predictor variables included primary strategy group and strategy repertoire group. Consistent with the analysis of blank-final problems, the effect of strategy group on the probability of making at least one conceptual error was significant when controlling for strategy repertoire group, Wald $(2, N = 67) = 5.91$, $p = .05$. However, the effect of strategy repertoire group was not significant when controlling for primary strategy group, Wald $(2, N = 67) = 0.14$.

Students in the correct strategy group were less likely than students in the incorrect strategy groups to have made at least one conceptual error in encoding non-blank-final problems

(8 of 24 vs. 24 of 43), $\beta = -2.764$, $z = 2.25$. The model estimates that the odds of making at least one conceptual error are more than 15 times lower for students in the correct group than for students in the incorrect groups. Students in the add-all group were more likely than students in the other-incorrect group to make at least one conceptual error (9 of 11 vs. 15 of 32), $\beta = 1.590$, $z = 1.84$, $p = .07$. The model estimates that odds of making at least one conceptual error are almost five times higher for students in the add-all group than for students in the other-incorrect group. However, this effect is only marginally significant, so it should be interpreted with caution.

4. Discussion

Our findings suggest that encoding is influenced by the extent to which problem structure matches previously learned perceptual patterns. Students' encoding was poorest on blank-final equivalence problems, which overlap with but do not conform to the pattern of having "$= \_$" at the end of the problem. Students sometimes assimilated blank-final equivalence problems to the pattern and encoded them as if they were typical addition problems. Thus, experience with particular patterns in a domain can hinder encoding performance when those patterns do not match patterns that are encountered subsequently.

It is worth highlighting that students' encoding depended on perceptual features of the problems, rather than on the conceptual structure of the problems. Conceptually, the blank-final problems are identical to the non-blank-final problems, and neither conforms to the preexisting category of typical addition problems. If a problem's conceptual relationship to preexisting categories dictated how it is encoded, one would expect students to encode typical addition problems accurately and both types of equivalence problems inaccurately. In the present study, students performed well on typical addition problems and poorly on blank-final equivalence problems. However, they performed well on non-blank-final equivalence problems. Thus, encoding did not depend on whether students had a preexisting category within which to classify the problems. Instead, encoding depended on perceptual features of the problems.

The present study revealed systematic relationships between encoding and strategy use. The findings are consistent with the classic finding that individuals who use correct strategies encode problems more accurately than individuals who use incorrect strategies. Unlike prior studies, however, we demonstrated this relationship in a sample in which strategy use is not confounded with age or domain experience. All of the participants were about the same age, and they had comparable domain experience, because all were in fourth grade. Moreover, the present study provided a means for distinguishing between two possible accounts of the classic finding: the freed-resources account and the action-primacy account.

Our results are consistent with the action-primacy account, which maintains that solvers encode only those problem features necessary for guiding action. Students who used the "add all" strategy to solve equivalence problems encoded the numbers more accurately, but the problem structure less accurately, than students who used other incorrect strategies. Students accurately encoded only those features of the problem necessary for executing their highly-practiced "add all" strategy, and they disregarded other problem features.

The observed relationship between the number of strategies in students' repertoires and encoding performance also supports the action-primacy account. For blank-final problems, students who considered a variety of strategies to solve the problems made fewer conceptual errors than did students who considered only a single strategy. Thus, heightened strategy variability is associated with more accurate encoding of problem structure. Problem solvers who use only a single strategy encode only those problem features relevant to that particular strategy, and they may fail to encode other problem features. In contrast, problem solvers who have many different strategies in their repertoires need to choose among those strategies. They may encode many problem features in an effort to choose among available strategies, or they may use the encoded problem features to construct a new strategy. Consistent with this view, past work (Goldin-Meadow, Alibali, & Church, 1993) has suggested that individuals who exhibit variability in their strategy use may be particularly open to encoding instruction-related information. Some researchers (e.g., Grunow & Neuringer, 2002) have suggested that variability itself increases an individual's sensitivity to environmental change and facilitates learning.

Although students who exhibited variability in their strategy use made fewer conceptual encoding errors, students who expressed only a single strategy made fewer number errors. This is also not surprising when considered in the context of the action-primacy account. When a solver has a single, highly-practiced strategy, he or she already has a strategy on hand, and thus, can quickly turn attention to encoding the problem features that are required for executing that strategy (e.g., the numbers).

Overall, the present findings support the action-primacy account. Nonetheless, the observed associations between encoding and strategy use are just that—associations. The present study cannot address whether differences in strategy use and strategy variability cause encoding differences or vice versa. Some past work has shown that changes in encoding lead to changes in strategy use (e.g., Alibali et al., 1998; Siegler, 1976); however, the action-primacy account suggests that the reverse may be possible in some cases. The likely resolution is that encoding and strategy use are tightly coupled, with small changes in one leading to small changes in the other in an iterative fashion (Rittle-Johnson, Siegler, & Alibali, 2001). The action-primacy account makes specific predictions about when encoding should inform strategy construction and when strategy knowledge should drive encoding. Specifically, when individuals do not have a highly-practiced strategy on hand, their intended action will be to devise a strategy. In these cases, individuals will encode features to use in the construction of new strategies. In contrast, when individuals have a highly-practiced strategy on hand, that strategy will dictate encoding, and they will encode only those features necessary for executing it. We are currently conducting experiments to test these hypotheses.

At least two implications for instruction can be drawn from the present findings. First, educators should be aware of the costs of drill on problems that do not directly map on to future endeavors. In the present study, a perceptual pattern and a strategy that students learned from their experience with arithmetic interfered with encoding of novel equations. This leads to the unintuitive prediction that students with stronger arithmetic skills may be more at risk for some types of difficulties at the transition to algebra. We are currently testing this prediction. At a minimum, when introducing new problem types, educators may wish to highlight explicitly how they differ structurally from familiar problems.

Second, most fourth-grade students do not have a good understanding of the syntax of equations. Conceptual errors were frequent, and many students omitted the equal sign altogether, even on typical addition problems. These findings converge with other work indicating that understanding of the equal sign is limited in children in this age range (Baroody & Ginsburg, 1983; Kieran, 1981; McNeil & Alibali, 2000). Understanding of equations should be a target for instruction in elementary school.

In sum, students use their knowledge of prior perceptual patterns as well as their knowledge of highly-practiced strategies to encode problems efficiently. The present study showed that (1) encoding suffers when problems overlap with, but do not directly correspond to, previously-learned perceptual patterns and (2) individual differences in strategy use and strategy variability are intimately related to variations in encoding. These results highlight that encoding is intended to guide action and that prior experience can simultaneously facilitate and interfere with accurate encoding. Thus, to understand what students notice in problem-solving situations, it is essential to consider what they have learned in the past.

Note

1. In keeping with past work (Chase & Simon, 1973; Siegler, 1976), we utilize performance on a reconstruction task as a measure of encoding. However, given that we do not have a looking time measure to corroborate this assumption, an equally plausible interpretation is that students encode the problems accurately and then use their knowledge of arithmetic patterns to distort the problems in the reconstruction process. Recently, we created a recognition version of the reconstruction measure of problem encoding to address this alternative hypothesis (McNeil & Alibali, 2000, 2002). Students are given a sheet of paper with seven problems on it. One of the problems is an equivalence problem in its correct form (e.g., $a + b + c = a + \_$). The other six problems depict errors students typically make when reconstructing equivalence problems, including the typical addition error ($a + b + c + a = \_$). After viewing an equivalence problem for 5 s, students are asked to find and circle the problem they just saw. We have found this measure to be highly correlated with the reconstruction measure used in the present study, suggesting that errors do not lie in the process of reconstruction itself.

Acknowledgments

This research was supported by Behavioral Sciences Track Award for Rapid Transition grant R03 MH55207-01 from the National Institutes of Mental Health and by a Faculty Development Award from Carnegie Mellon University, both to Martha W. Alibali. Portions of these data were presented at the 1998 meeting of the Jean Piaget Society and at the 2001 meeting of the Society for Research in Child Development. We thank Audrey Russo, Michael Perrott, and Leslie Petasis for assistance with data collection and coding, and Julia Evans, Jerry Haeffel, Chuck Kalish, Ken Kotovsky, Bethany Rittle-Johnson, and Vladimir Sloutsky for helpful discussions and feedback on previous versions of the paper. We extend special thanks to the students, parents, teachers and administrators at St. Maurice, Word of God, Sacred Heart and Carlow Campus schools in the Diocese of Pittsburgh, who made this research possible.

References

Alibali, M. W. (1999). How children change their minds: Strategy change can be gradual or abrupt. *Developmental Psychology*, *35*, 127–145.

Alibali, M. W., McNeil, N. M., & Perrott, M. A. (1998). What makes children change their minds? Changes in encoding lead to changes in strategy selection. In M. A. Gernsbacher & S. Derry (Eds.), *Proceedings of the Twentieth Annual Conference of the Cognitive Science Society*. Mahwah, NJ: Lawrence Erlbaum.

Baroody, A. J., & Ginsburg, H. P. (1983). The effects of instruction on children's understanding of the "equals" sign. *Elementary School Journal*, *84*, 199–212.

Beaton, A. E., Mullis, I. V. S., Martin, M. O., Gonzalez, E. J., Kelly, D. L., & Smith, T. A. (1996). *Mathematics achievement in the middle school years: IEA's Third International Mathematics and Science Study (TIMSS)*. Chestnut Hill, MA: Boston College.

Best, C. T., McRoberts, G. W., & Goodell, E. (2001). Discrimination of non-native consonant contrasts varying in perceptual assimilation to the listener's native phonological system. *Journal of the Acoustical Society of America*, *109*, 775–794.

Bruner, J. S. (1957). On perceptual readiness. *Psychological Review*, *2*, 123–152.

Chase, W. G., & Simon, H. A. (1973). Perception in chess. *Cognitive Psychology*, *4*, 55–81.

Chi, M. T. H. (1978). Knowledge structures and memory development. In R. S. Siegler (Ed.), *Children's thinking: What develops?* (pp. 73–96). Hillsdale, NJ: Lawrence Erlbaum.

Chi, M. T. H., Feltovich, P. J., & Glaser, R. (1981). Categorization and representation of physics problems by experts and novices. *Cognitive Science*, *5*, 121–152.

Dean, A. L., & Malik, M. M. (1986). Representing and solving arithmetic word problems: A study of developmental interaction. *Cognition and Instruction*, *3*, 211–227.

Flege, J. E., Yeni Komshian, G. H., & Liu, F. (1999). Age constraints on second-language acquisition. *Journal of Memory and Language*, *41*, 78–104.

Fowler, C. A. (1986). An event approach to the study of speech perception from a direct-realist perspective. *Journal of Phonetics*, *14*, 3–28.

Gibson, J. J. (1979). *The ecological approach to visual perception*. Boston, MA: Houghton-Mifflin.

Goldin-Meadow, S., Alibali, M. W., & Church, R. B. (1993). Transitions in concept acquisition: Using the hand to read the mind. *Psychological Review*, *100*, 279–297.

Goldin-Meadow, S., Nusbaum, H., Garber, P., & Church, R. B. (1993). Transitions in learning: Evidence for simultaneously activated strategies. *Journal of Experimental Psychology: Human Perception and Performance*, *19*, 92–107.

Grunow, A., & Neuringer, A. (2002). Learning to vary and varying to learn. *Psychonomic Bulletin and Review*, *9*, 250–258.

Kaplan, C. A., & Simon, H. A. (1990). In search of insight. *Cognitive Psychology*, *22*, 374–419.

Kieran, C. (1981). Concepts associated with the equality symbol. *Educational Studies in Mathematics*, *12*, 317–326.

Knoblich, G., Ohlsson, S., Haider, H., & Rhenius, D. (1999). Constraint relaxation and chunk decomposition in insight problem solving. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *25*, 1534–1555.

Kotovsky, K., Hayes, J. R., & Simon, H. A. (1985). Why are some problems hard? Evidence from the Tower of Hanoi. *Cognitive Psychology*, *17*, 248–294.

Kuhl, P. K. (2000). A new view of language acquisition. *Proceedings of the National Academy of Science*, *97*, 11850–11857.

Luchins, A. S., & Luchins, E. H. (1959). *Rigidity of behavior: A variational approach to the effect of Einstellung*. Eugene, OR: University of Oregon Press.

McNeil, N. M., & Alibali, M. W. (2000). Learning mathematics from procedural instruction: Externally imposed goals influence what is learned. *Journal of Educational Psychology*, *92*, 734–744.

McNeil, N. M., & Alibali, M. W. (2002). A strong schema can interfere with learning: The case of children's typical addition schema. In C. D. Schunn & W. Gray (Eds.), *Proceedings of the Twenty-Fourth Annual Conference of the Cognitive Science Society*. Mahwah, NJ: Lawrence Erlbaum.

Morales, R. V., Shute, V. J., & Pellegrino, J. W. (1985). Developmental differences in understanding and solving simple mathematics word problems. *Cognition and Instruction*, *2*, 41–57.

Perry, M. (1988). Problem assignment and learning outcomes in nine fourth-grade mathematics classes. *Elementary School Journal*, *88*, 413–426.

Perry, M., Church, R. B., & Goldin-Meadow, S. (1988). Transitional knowledge in the acquisition of concepts. *Cognitive Development*, *3*, 359–400.

Rittle-Johnson, B., Siegler, R. S., & Alibali, M. W. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. *Journal of Educational Psychology*, *93*, 346–362.

Rumelhart, D. E. (1980). Schemata: The building block of cognition. In R. Spiro, B. Bruce, & W. Brewer (Eds.), *Theoretical issues in reading comprehension* (pp. 33–58). Hillsdale, NJ: Lawrence Erlbaum.

Shiffrin, R. M., & Schneider, W. (1977). Controlled and automatic human information processing. II. Perceptual learning, automatic attending, and a general theory. *Psychological Review*, *84*, 127–190.

Shrager, J., & Siegler, R. S. (1998). SCADS: A model of children's strategy choices and strategy discoveries. *Psychological Science*, *9*, 405–410.

Siegler, R. S. (1976). Three aspects of cognitive development. *Cognitive Psychology*, *8*, 481–520.

Siegler, R. S. (1989). Mechanisms of cognitive development. *Annual Review of Psychology*, *40*, 353–379.

Siegler, R. S., & Chen, Z. (1998). Developmental differences in rule learning: A microgenetic analysis. *Cognitive Psychology*, *36*, 273–310.

Valverde, G. A., & Schmidt, W. H. (1997, Winter). Refocusing U.S. math and science education. *Issues in Science and Technology*.

Zevin, J. D., & Seidenberg, M. S. (2002). Age of acquisition effects in reading and other tasks. *Journal of Memory and Language*, *47*, 1–29.