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Why Won't You Change Your Mind? Knowledge of Operational Patterns Hinders Learning and Performance on Equations

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This study examined whether knowledge of arithmetic contributes to difficulties with equations. In Experiment 1, children (ages 7–11) completed tasks to assess their adherence to 3 operational patterns prevalent in arithmetic: (a) the strategy of performing all given operations on all given numbers, (b) the "operations = answer" problem structure, and (c) the concept that the equal sign means "the total." Next, children received a lesson on equations; then, they solved a set of equations. There was a negative relationship between adherence to the operational patterns and learning. In Experiment 2, undergraduates' knowledge of the operational patterns was activated or not. Students whose knowledge was activated did not perform as well on equations. Results suggest that early-learned patterns constrain future learning and performance.

A central issue in the study of cognitive development is how change occurs (Siegler, 2000). Research addressing this issue has yielded beautifully detailed accounts of the path, rate, breadth, and variability of cognitive change (e.g., Adolph, 1997; Dixon & Bangert, 2002; Gershkoff-Stowe, 2002; Siegler, 1989; Siegler & Stern, 1998). However, some ways of thinking resist change, even after substantial amounts of training or instruction. Indeed, some domains of knowledge (e.g., mathematics, science, foreign language) are so difficult to learn that many people fail to achieve basic competence, even after years of schooling. Consequently, any theory of learning must explain not only how people change, but also why people resist change. We examined change resistance in the domain of mathematics, focusing on children's difficulties learning about mathematical equations.

An equation is any mathematical statement that uses the equal sign to indicate that two mathematical expressions are (or are defined to be) equivalent. Many studies have shown that elementary school children (ages 7–11) have difficulties solving equations, especially equations with operations on both sides of the equal sign (e.g., $7+4+5=7+\_\_$; Carpenter, Franke, & Levi, 2003; Perry, Church, & Goldin-Meadow, 1988). In the absence of instruction, approximately 75% of third- through fifth-grade children in schools with traditional mathematics curricula solve such equations incorrectly (Alibali, 1999; McNeil & Alibali, 2000). Even after receiving instruction, children have limited transfer and poor retention of correct strategies (Alibali, 1999; McNeil & Alibali, 2000; Perry, 1991; Rittle-Johnson & Alibali, 1999). Children also judge equality statements with operations on both sides of the equal sign (e.g., $3+4=5+2$) as incorrect, nonsensical, or false (Baroody & Ginsburg, 1983; Behr, Erlwanger, & Nichols, 1980; Carpenter et al., 2003).

The mechanisms underlying children's difficulties with equations and the ultimate emergence of correct strategies are not well understood. Many theories suggest that difficulties are due to something that children lack. For example, children between ages 7 and 11 are said to lack domain-general logical structures, such as the ability to coordinate relationships of equivalence or to view an operation as an object (Piaget & Szeminska, 1941/1995; Sfard & Linchevski, 1994). Children are also said to lack a mature working memory system, whether working memory is defined in terms of total capacity (Cowan, Nugent, Elliot, Ponomarev, & Saults, 1999; Pascual-Leone, 1970), efficiency (Case, Kurland, & Goldberg, 1982), or processing speed (Hulme, Thomson, Muir,

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& Lawrence, 1984; Kail & Park, 1994; Kail & Salthouse, 1994). Children are also said to lack proficiency with basic arithmetic operations, which is assumed to be necessary for advanced thinking and problem solving in mathematics (Anderson, 2002; Haverty, 1999; Kotovsky, Hayes, & Simon, 1985). Thus, according to several prevailing accounts, equation-learning difficulties are due to something that children lack.

In contrast to this focus on what children lack, some scientists (e.g., Flege, Yeni Komshian, & Liu, 1999; Kuhl, 2000; McNeil & Alibali, 2002, 2004; Schauble, 1990; Zevin & Seidenberg, 2002) have considered the role of existing knowledge in learning difficulties. The general theoretical claim is that early learning constrains later learning. This view is derived from classic approaches to cognition and perception that focus on top-down processes (e.g., Bruner, 1957; Rumelhart, 1980; Tolman, 1948). Contemporary theorists (e.g., Zevin & Seidenberg, 2002) explain the effects in terms of entrenchment of early-learned patterns. According to this view, the patterns with which people initially gain experience become entrenched, and learning difficulties arise when to-be-learned information overlaps with, but does not map directly onto, entrenched patterns.

Within this framework, we propose the change-resistance account of children's equation-learning difficulties. This account posits that children's equation-learning difficulties are due, at least in part, to their early and elongated experience with arithmetic operations (McNeil & Alibali, 2002). According to this view, excessive practice with arithmetic operations hinders subsequent learning of more complex equations. Thus, in contrast to accounts that focus on what children lack, the change-resistance account focuses on what children have.

Children's early mathematics experience in the United States is dominated by arithmetic operations (Baroody & Ginsburg, 1983; Beaton et al., 1996; McNeil et al., 2004; Valverde & Schmidt, 1997). Children encounter the same operational patterns repeatedly, virtually without exception (Seo & Ginsburg, 2003). As a result, children's internal representations of these patterns gain strength, and children develop what Hatano (1988) termed "routine expertise" with arithmetic operations. Children are proficient at applying what they have learned (patterns and procedures), but they may not understand the concepts that underlie what they have learned (Hatano, 1988). As a result, children apply their knowledge of the operational patterns inflexibly and do not generate new strategies when they encounter novel equations.

We argue that children learn three operational patterns that ultimately hinder their ability to learn more complex equations. First, children learn the equation-solving strategy "perform all given operations on all given numbers." For example, when presented with a typical addition problem such as $2+3+4+5=\_\_$, children add all the numbers and put the total, 14, in the blank (McNeil & Alibali, 2000; Perry et al., 1988). Second, children learn a perceptual pattern related to the structure of equations, namely the "operations = answer" structure (McNeil & Alibali, 2004; see also Seo & Ginsburg, 2003). In a typical addition problem such as $2+3+4+5=\_\_$, the numbers and operations are to the left of the equal sign, and the answer blank is to the right. Third, children learn a particular concept of the equal sign. Specifically, they infer that the equal sign means "the total" (McNeil & Alibali, in press; see also Baroody & Ginsburg, 1983; Behr et al. 1980; Kieran, 1981). Note that our use of the term *operational* differs from the Piagetian use of the term. We refer to these patterns as operational patterns because children learn them from their experience with arithmetic operations.

The change-resistance account holds that the entrenchment of operational patterns contributes to children's difficulties with equations. When children encounter novel equations, their representations of the operational patterns are activated, and these representations control attention, determine what is encoded, and influence how information is interpreted. This view is compatible with accounts of how knowledge influences attention and performance in other domains (Bruner, 1957; Chase & Simon, 1973; Chi, 1978; Knoblich, Ohlsson, Haider, & Rhenius, 1999; Knoblich, Ohlsson, & Raney, 2001; Samuelson, 2001). However, it challenges accounts of problem solving that imply that children need more practice with arithmetic operations to solve complex equations successfully (e.g., Anderson, 2002; Anderson, Reder, Lynne, & Lebiere, 1996; Haverty, 1999; Kotovsky et al., 1985).

Entrenchment accounts such as the change-resistance account are closely related to the classic phenomenon of mental set, in which a familiar, well-practiced approach to solving a problem can interfere with subsequent problem solving (Luchins, 1942; Wiley, 1998). Luchins (1942) illustrated this phenomenon in his famous water jar experiments. After solving a set of problems using a multistep strategy, participants failed to use an efficient, single-step strategy on a similar problem. The more problems solved with the multistep strategy, the more likely participants were to use the multistep strategy on the single-step problem.

The change-resistance account goes beyond Luchins's (1942) work in three ways. First, it posits that mental sets can be constructed and applied not only within a single problem-solving experience, as in Luchins's experiments, but also over the course of development (McNeil & Alibali, 2000). Children's knowledge of operational patterns is constructed over the course of elementary school, and children may apply that knowledge any time they encounter an equation. Second, the change-resistance account argues that mental sets are applied not only at the expense of efficiency, as in Luchins's experiments, but also at the expense of correctness (McNeil & Alibali, 2004). Children may adhere to their knowledge of operational patterns even when it does not lead to correct solutions. Third, the change-resistance account suggests that mental sets can interfere not only with performance, as in Luchins's experiments, but also with learning. We argue that knowledge of operational patterns hinders children's ability to learn from instruction on novel equations.

The change-resistance account is also compatible with claims that strategy variability is an impetus for cognitive change (e.g., Alibali, 1999; Alibali & Goldin-Meadow, 1993; Siegler, 1989, 1994). Indeed, it seems likely that entrenchment and strategy variability are negatively correlated. The more experience and practice a child has with a particular strategy, the more entrenched that strategy will be, and the less likely the child will be to exhibit strategy variability across problems. For example, as children gain experience with the operation of addition, they proceed from using a variety of addition strategies to relying primarily on a single strategy—retrieval (Shrager & Siegler, 1998; Siegler, 1987).

However, it is important to bear in mind that less variability does not necessarily mean deeper entrenchment. Consider an example from McNeil and Alibali (2000). Children were presented with an equation such as $7+4+5=7+\_\_$ and instructed to solve it using the grouping strategy (i.e., cancel the two 7s and add $4+5$). Immediately afterward, children were presented with a set of similar equations, and most children used the grouping strategy to solve all of them. Two weeks later, children were again presented with a set of similar equations, but this time, many children did not use the grouping strategy. In this example, children did not display strategy variability after learning the grouping strategy, but the grouping strategy was not entrenched. Thus, although entrenchment and lack of variability may be related, they are not synonymous. Because the change-resistance account focuses specifically on the role of entrenched knowledge in learning and performance difficulties, we controlled for potential effects of strategy variability in the present work.

Past work supports the view that entrenched knowledge of operational patterns contributes to difficulties with equations. Elementary school children (ages 7–11) have been shown to adhere to the operational patterns when they encounter novel equations. For example, when asked to solve equations such as $7+4+5=7+\_\_$, most children adhere to the strategy of performing all given operations on all given numbers (i.e., the add-all strategy in this context) and put 23 in the blank (McNeil & Alibali, 2000, 2002, 2004). When asked to reconstruct the equation after viewing it briefly, many children adhere to the "operations = answer" problem structure and write $7+4+5+7=\_\_$ (McNeil & Alibali, 2002, 2004). When asked to define the equal sign, many children adhere to the operational equal sign concept and say, "It means the total the total" (McNeil & Alibali, in press). Thus, there are data to suggest that children's performance on novel equations suffers because they adhere to the operational patterns.

The change-resistance account predicts not only that children will adhere to the operational patterns when they encounter novel equations, but also that children's adherence to the operational patterns contributes to their learning difficulties. However, the role of change resistance in learning has yet to be tested. No evidence has addressed whether knowledge of the operational patterns hinders children's ability to learn from instruction on equations.

Equally important, although past work has demonstrated a relationship between adherence to the operational patterns and poor performance on equations such as $7+4+5=7+\_\_$, all of the evidence has been correlational. The change-resistance account posits that adherence to the operational patterns causes poor performance on equations. Understanding the causes of poor performance is important because of the potential role of performance in learning and cognitive change (e.g., Sophian, 1997). Each time a problem is solved, the results may feed back to influence the solver's knowledge about that class of problems (e.g., McClelland, 1995; Shrager & Siegler, 1998). Thus, understanding the causes of poor performance on equations may reveal the roots of children's equation-learning difficulties.

In this article we present two experiments designed to test the relationship between knowledge of operational patterns and difficulties with equations. In the first experiment, we used an individual differences approach to investigate whether children's adherence to the operational patterns influences whether they learn from instruction on equations.

This experiment is the first to investigate the relationship between knowledge of arithmetic operations and difficulties learning about equations. In the second experiment, we tested the potential causal link between knowledge of operational patterns and difficulties solving equations. We did so by activating participants' knowledge of the operational patterns and assessing their equation-solving performance. Again, it is important to note that many theories (and educators) argue that practice and proficiency with basic arithmetic operations is the key to improving children's achievement on complex equations. According to this view, activating people's knowledge of arithmetic operations should facilitate equation-solving performance.

Experiment 1

The purpose of Experiment 1 was to investigate the relationship between amount of adherence to the operational patterns and likelihood of learning from a lesson on equations. To this end, we first measured individual differences in adherence to the operational patterns. Then, we gave children one of four brief lessons on equations. The lessons were based on those used in previous research (Alibali, McNeil, & Perrott, 1998; Rittle-Johnson & Alibali, 1999) that have been shown to remedy children's adherence to the operational perceptual pattern (i.e., "operations = answer" problem structure) and the operational concept of the equal sign (i.e., the equal sign means "the total"). Finally, we examined which (if any) children learned from the brief lessons by testing their equation-solving performance. We operationalized learning as the ability to used the knowledge gained from the lessons to generate new strategies for solving equations. The change-resistance account posits that children who adhere most to the operational patterns should be least likely to generate new equation-solving strategies.

Method

Participants

Participants were 91 children ranging in age from 7 years 10 months to 11 years 2 months ($M = 9$ years 2 months). Children were recruited from six schools (three public, three parochial) in the greater Madison, Wisconsin, area. The targeted schools spanned a broad range of socioeconomic levels as indicated by the percentage of children receiving free or reduced lunch in the three public schools (57%, 25%, and 11%). Because the purpose of the experiment was to predict individual differences in learning, 24 children (M age = 9 years 5 months) were excluded from the sample because they solved at least one of the equations on the equation-solving measure (described later) correctly before the lesson. Of the 24 children who solved at least one of the equations correctly before the lesson, 1 (4%) was from the public school with the lowest socioeconomic status, 8 (33%) were from the public school with intermediate socioeconomic status, and 9 (37.5%) were from the public school with the highest socioeconomic status. The remaining 6 were from the three parochial schools.

The final sample consisted of 67 children (29 boys, 38 girls; 6 African American, 61 Caucasian) ranging in age from 7 years 10 months to 11 years 2 months (M age = 9 years 0 months) who solved a set of equations with operations on both sides of the equal sign incorrectly before the lesson. It is important to note that children in this age range are very familiar with the operation of addition. Most children can solve addition problems correctly (with the use of informal strategies) when they enter kindergarten around the age of 5 (Geary & Burlingham-Dubree, 1989; Siegler & Shrager, 1984), and after children enter formal schooling, they receive a great deal of practice with the operation of addition. Thus, it is reasonable to assume that the children in this study were experienced addition problem solvers.

Measures

Equation solving. The equation-solving measure consisted of three equations with operations on both sides of the equal sign (e.g., $7+4+5=7+\_\_$). We chose equations of this type because past work has shown that children (ages 7–11) find such equations difficult (e.g., McNeil & Alibali, 2004). The experimenter placed each equation on an easel and said, "Try to solve the problem as best as you can, and then put your answer in the blank." After children wrote their solutions, the experimenter said, "Can you tell me how you got x?" (with x symbolizing the solution). After explaining each solution, children were asked to rate how certain they were about their "way of doing the problem" on a 7-point scale that ranged from 1 (*it's definitely wrong*) to 7 (*it's definitely right*), with 4 (*I'm not sure if it's right or wrong*) as the midpoint.

Problem structure. Two tasks made up the problem structure measure. The first was taken from Rittle-Johnson and Alibali (1999). Children were asked to reconstruct three equations with operations on both sides of the equal sign after viewing each for 5 s. The second task was a recognition version of the first task; it also contained three equations with operations on both sides of the equal sign. Before viewing each equation, children were given a sheet of paper face down with seven equations on it. One of the equations on the sheet matched the equation that children would be shown. The other six equations depicted errors that children typically make when reconstructing equations with operations on both sides of the equal sign, including the "operations = answer" foil (e.g., for the equation 7+4+5 = 7+__, the "operations = answer" foil would be 7+4+5+7 = __). After viewing an equation for 5 s, children were instructed to turn the sheet of paper over, find the problem they just saw, and circle it. Children repeated this process for all three equations.

Equal sign definition. Two tasks made up the equal sign definition measure. Both were taken from Rittle-Johnson and Alibali (1999). In the first task, the experimenter presented an equation with operations on both sides and pointed to the equal sign as she said, "I want you to tell me what you think this math symbol means." In the second task, children were asked to rate the "smartness" of six fictitious students' definitions as "not so smart," "kind of smart," or "very smart." The definitions were: "the answer to the problem," "repeat the numbers," "the end of the problem," "something is equal to another thing," "two amounts are the same," and "the total."

Procedure

Children participated individually in one videotaped session conducted by a female experimenter in a quiet room during school hours. Children first completed the equation-solving measure, followed by the problem structure and equal sign definition measures in random order. Then, children were randomly assigned to one of four brief lesson conditions in a 2 (problem structure lesson or no problem structure lesson) $\times$ 2 (equal sign concept lesson or no equal sign concept lesson) factorial design. During the intervention, children in all four conditions were presented with an equation with the correct solution in the blank (6+4+7 = 6+$\underline{11}$). In the control condition, children were shown the correctly solved equation and told that it was a correctly solved equation. They were encouraged to think about the correctly solved equation for 1 min. Children in the other three conditions were also presented with the correctly solved equation, told that it was a correctly solved equation, and encouraged to think about it. In addition, children who received the problem structure lesson were told to "notice where the equal sign is in the problem," and they were asked to point to the equal sign (cf. Alibali et al., 1998). Children who received the equal sign concept lesson were told to "notice that the equal sign means that the things on one side of it have to be the same amount as the things on the other side of it," and they were asked to repeat the definition (cf. Rittle-Johnson & Alibali, 1999). All of the lessons were brief, lasting a total of 1 min (timed with a timer). After the lesson, there was a manipulation check in which children completed the problem structure and equal sign definition measures in random order. The manipulation check was followed by the equation-solving measure. Finally, children solved four transfer problems, two that differed from the equations on the equation-solving measure in terms of the position of the blank (e.g., 6+5+8 = __+8) and two that differed in that they did not include a repeated addend (e.g., 4+3+7 = 5+__).

Coding

Equation solving. Strategies on the equation-solving measure were coded using the system developed by Perry et al. (1988). Strategies were assigned based on children's problem solutions and verbal explanations. The most common strategy was the add-all strategy (e.g., for 7+4+5 = 7+__ , writing 23 in the blank and saying "I added up all the numbers"). The add-all strategy was used on 87% of equations before the lesson. Children rarely used other incorrect strategies: Three percent of the equations were solved using the carry strategy (e.g., writing 4 in the blank and saying "There was a 4 here, so I put 4"), 8% were solved using the add-to-equal-sign strategy (e.g., writing 16 in the blank and saying "I added seven plus four plus five"), and 2% were solved using other idiosyncratic strategies. Age was not correlated with use of the add-all strategy before the lesson ($r = -.097$, $p = .45$).

Children's certainty ratings about their way of doing the problem were used to categorize children according to whether they thought they used a correct strategy. Recall that certainty ratings were made using a scale that ranged from 1 (*it's definitely wrong*) to 7 (*it's definitely right*), with 4 (*I'm not sure if it's right or wrong*) as the midpoint. Age was not correlated with average certainty rating ($r = -.082$, $p = .53$).

Problem structure. Children's reconstructions were coded using a system developed by McNeil and Alibali (2004). Each reconstruction was examined for conceptual errors. Conceptual errors involve inaccurate reconstructions of the problem structure, such as omitting the equal sign or one of the plus signs (e.g., for the equation 7+4+5 = 7+__, writing 7+4+5 = 7__). Cases in which the right addend was omitted altogether and cases in which equations were converted to the traditional ''operations = answer'' problem structure were also classified as conceptual errors. Errors in reconstructing the particular numbers or order of numbers were not considered conceptual errors (e.g., for the problem 7+4+5 = 7+__, writing 4+7+5 = 7+__). Reconstructions were coded as correct if they were free of conceptual errors. On the recognition task, each of the three responses was scored as correct or incorrect based on whether children circled the correct equation on the sheet provided. Performance on the recall and recognition tasks was correlated ($r = .60$, $p < .001$).

Equal sign definitions. Children's definitions were coded according to a system developed by McNeil and Alibali (in press). Definitions were first coded as relational (e.g., ''two amounts are the same'') or not. Definitions that were not relational were further examined for whether they conveyed an arithmetic operation such as addition (e.g., ''the total'') or not (e.g., ''the end of the problem''). None of the children gave a relational definition before the brief lesson. Children's ratings of the fictitious students' definitions of the equal sign were converted to numerical scores. Two points were given for ''very smart'' ratings, 1 point was given for ''kind of smart'' ratings, and 0 points were given for ''not so smart'' ratings. As a manipulation check, the average rating for the incorrect definitions was subtracted from the average rating for the relational definitions to yield a difference score. A positive difference score in this case indicates that relational definitions such as ''two amounts are the same'' and ''something is equal to another thing'' were rated as smarter than less sophisticated definitions. On the whole, children's definitions and ratings were compatible with one another. For example, only 5 children rated the definition *the total* as ''not so smart,'' and none of these 5 children expressed the idea of adding, totaling, or summing in their own definitions.

Adherence to the operational patterns. We assessed children's adherence to the three operational patterns on the prelesson measures. On the equation-solving measure, children were coded as adhering to the operational strategy if they (a) used the add-all strategy on at least two of three equations (see Table 1) and (b) gave the add-all strategy an average certainty rating greater than 4 (on the 7-point scale). Recall that ratings of 4 or less indicate that children do not think their strategy is correct. Children who use the add-all strategy, but do not rate it as correct, are likely operating according to a back-up strategy rather than an entrenched pattern (see Siegler, 1983). On the problem structure measure, children were coded as adhering to the operational pattern if they showed evidence of converting at least two equations to typical addition problems (on either the reconstruction task or the recognition task; see Table 1). Recall that children exhibited a variety of incorrect ways of thinking on the problem structure measure, only one of which involved converting the equations to typical addition problems. On the equal sign definition measure, children were coded as adhering to the operational pattern if they showed evidence of thinking that the equal sign represents an arithmetic operation, such as addition. Children could show this in one of two ways. First, they could express the idea of adding, totaling, or summing in the definitions they provided (see Table 1). Or, alternatively, they could rate the definition *the total* as very smart. Recall that none of the children gave a relational definition of the equal sign before the lesson. To establish reliability in evaluating adherence to the operational patterns, a second coder recoded the data for 10 participants. Reliability was 100%.

Children were assigned a score based on the number of prelesson measures (out of three) on which they adhered to the operational patterns. Thus, children's operational pattern scores ranged from 0 to 3. The number of measures on which the

Table 1
Examples of Responses on Each Measure That Would Be Coded as Adhering or Not Adhering to the Operational Patterns for the Equation 7+4+5 = 7+__

Code	Problem solving (solution ‖ explanation)	Problem structure	Equal sign definition
Adheres	23 ‖ ''I added them all up.''	7+4+5+7 = __	''Add up all those together.''
Adheres	22 ‖ ''I added 7 plus 4 plus 5 plus 7.''	7+4+5+7 =	''The total of the problem.''
Does not adhere	4 ‖ ''There was a 4 here, so I put 4.''	7+4+5 = 7 = __	''What the problem is.''
Does not adhere	1 ‖ ''I just guessed.''	7+4+5 = +7__	''It's like where you end the problem.''

Note. All responses are incorrect.

pattern was exhibited was considered to reflect the extent of entrenchment of children's knowledge of the operational patterns (i.e., 3 reflects the greatest entrenchment).

Table 1 presents examples of responses that would and would not be coded as adhering to the operational patterns for each measure. It is important to emphasize that all children solved the problems incorrectly before the brief lesson, regardless of whether they adhered to the operational patterns. Moreover, there is no reason to believe that the thinking of children who do not adhere to the operational patterns is closer to correct than is the thinking of children who do adhere to the operational patterns. For example, a child who writes *1* in the blank when presented with the equation 8+7+6 = 8+__ is just as wrong as a child who writes *29* in the blank. Similarly, a child who reconstructs the equation 3+4+5 = 3+__ as 3+4+5 = +3 __ is just as wrong as a child who reconstructs the equation as 3+4+5+3 = __. Thus, outside of the change-resistance account, there is no reason to expect learning differences between children who do or do not adhere to the operational patterns.

Results and Discussion

Manipulation Check

We examined children's performance on the problem structure and equal sign definition measures both before and after the lessons. Our goal was to determine whether the brief lessons provided children with new, correct ways of thinking about the problem structure and the concept of the equal sign. A 2 (problem structure lesson or no problem structure lesson) $\times$ 2 (equal sign concept lesson or no equal sign concept lesson) analysis of covariance (ANCOVA) was performed with age (in days) as a covariate and pre- to post-lesson change in number correct on the problem structure measure as the dependent variable. As expected, the analysis revealed a significant main effect for the problem structure lesson, $F(1, 63) = 4.14$, $p = .04$, $\eta_p^2 = .068$. Children who received the problem structure lesson improved their performance on the problem structure measure after the lesson ($M = 1.78$, $SD = 1.31$) more than did children who did not receive the problem structure lesson ($M = 1.00$, $SD = 1.65$). None of the other effects was significant (all $F\text{s} < 1$). A similar 2×2 ANCOVA was performed with pre- to post-lesson change in difference score on the smartness ratings portion of the equal sign definition measure as the dependent variable. Recall that a positive difference score indicates that children rated relational definitions such as "two amounts are the same" as smarter than less sophisticated definitions such as "the end of the problem." As expected, the analysis revealed a significant main effect for equal sign concept lesson, $F(1, 63) = 31.41$, $p < .001$, $\eta_p^2 = .35$. Children who received the equal sign concept lesson improved their smartness ratings after the lesson ($M = 0.89$, $SD = 0.86$) more than did children who did not receive the equal sign concept lesson ($M = -0.08$, $SD = 0.50$). None of the other effects was significant; for the main effect of problem structure lesson, $F(1, 63) = 1.61$, $p = .21$; all other $F\text{s} < 1$.

The manipulation checks indicate that, as in previous work (Alibali et al., 1998; Rittle-Johnson & Alibali, 1999), the brief lessons imparted the intended information about equations. Children were, in general, able to take in the information in its specific form. The main question, however, is whether children really learned from the lessons. That is, did they use the knowledge gained from the brief lessons to generate new strategies for solving equations? Furthermore, did they generate correct problem-solving strategies?

Effects of Adherence to the Operational Patterns

Recall that all children solved the equations incorrectly before the brief lesson. Furthermore, children's operational pattern score (0–3) was independent of instruction condition (Fs for main effects and interaction < 1). Our main question was whether children's adherence to the operational patterns influenced strategy change after the brief lessons. To foreshadow the results, the lessons themselves did not predict changes in strategy use. This is not surprising, given that (a) all four of the lesson conditions provided at least some new, correct information about the equations that children could use in constructing correct equation-solving strategies, and (b) we predicted individual differences in learning based on the extent to which children adhered to the operational patterns before the lessons.

Consistent with prior work (e.g., Alibali, 1999; Alibali et al., 1998), children were classified as generating a strategy if they solved any of the postlesson equations using a different strategy than they had used to solve the prelesson equations. However, it is important to note that we cannot be entirely certain that children actually generated new strategies after the lesson. It is possible that children had the strategies in their repertoires before the lesson but did not use them until after the lesson. We use the term *generate* for conciseness throughout this article, but we acknowledge that it is possible that our findings reflect changes in children's strategy choices rather than strategy generation.

We used logistic regression to examine the relationship between operational pattern score (0–3) and strategy generation (generate or not). Recall that the change-resistance account predicts a negative linear relationship between operational pattern category and strategy generation. We also included the two lesson conditions (problem structure lesson and equal sign concept lesson) and their interaction, as well as age (in days) as predictors in the model.

As predicted by the change-resistance account, there was a significant negative linear relationship between operational pattern score (0–3) and strategy generation (generate or not) when controlling for the other predictors in the model, $\hat{\beta} = -1.15$, $z = -2.66$, Wald $(1, N = 67) = 7.09$, $p = .008$. None of the other predictors was significant (problem structure lesson: $\hat{\beta} = -1.38$, $z = -1.63$; equal sign concept lesson: $\hat{\beta} = -0.94$, $z = -1.13$; interaction of lesson conditions: $\hat{\beta} = 1.29$, $z = 1.09$; age: $\hat{\beta} = 0.002$, $z = 1.58$).

Figure 1 displays the relationship between operational pattern score and the proportion of children who generated a strategy after the brief lesson. The model estimates that the odds of generating a strategy decrease by 3.15 for each unit increase in adherence to the operational patterns. Thus, the odds of strategy generation are nearly 9.5 times lower for children who adhere to all three operational patterns than for children who do not adhere to any of the operational patterns. Very few (1 of 9) children who adhered to the operational patterns on all three measures generated a strategy after the lesson, whereas all (5 of 5) children who did not adhere to the operational patterns on any of the measures did. Such a high proportion of strategy generation by children who did not adhere to the operational patterns is surprising, given that the lessons lasted for only 1 min. A moderate proportion of children who adhered to the operational patterns on one or two measures generated a strategy (14 of 22 and 13 of 31, respectively). Again, all children used incorrect strategies before the intervention; therefore, it was adherence to the operational patterns per se that was associated with strategy generation after the lessons.

More detailed information about the relationship between adherence to the operational patterns and strategy generation is presented in Table 2, which displays the number of children who adhered to the operational patterns on each of the three measures, along with the proportion of children in each cell who generated a strategy. On the whole, children who adhered to the operational pattern on a given measure were less likely than children who did not adhere to the operational pattern on that measure to generate a strategy (equal sign definition: *M*s = .55 vs. .72; problem solving: *M*s = .35 vs. .92; problem structure: *M*s = .62 vs. .65).

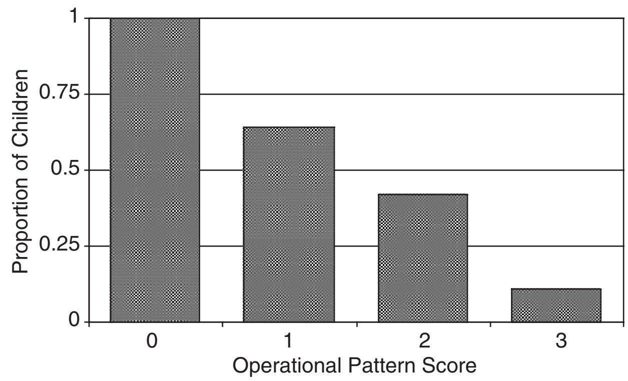

Figure 1. Proportion of children who used a new strategy after the lesson as a function of operational pattern score.

We next examined whether children generated correct strategies after the lesson. Children were classified as generating a correct strategy if they solved any of the three postlesson equations using a correct strategy. For example, children would be classified as generating a correct strategy for the equation 7+4+5 = 7+__ if they put the solution *9* in the blank and said that they added 4 plus 5 to get 9.

We used logistic regression to examine the relationship between operational pattern score (0–3) and use of a correct strategy (correct or not). The predictor variables in the model were identical to those used in the previous logistic regression model. As predicted by the change-resistance account, there was a significant negative linear relationship between operational pattern score (0–3) and use of a correct strategy (correct or not) when controlling for

Table 2
Proportion of Children in Each of the Possible Combinations of Operational Pattern Adherence on the Equal Sign Definition Measure (D), the Equation-Solving Measure (S), and the Problem-Structure Measure (R) Who Generated a Strategy After the Lesson

D adheres	S adheres	S does not adhere
R adheres	.11 ($n = 9$)	1.0 ($n = 1$)
R does not adhere	.41 ($n = 22$)	.70 ($n = 10$)
D does not adhere	**S adheres**	**S does not adhere**
R adheres	.375 ($n = 8$)	1.0 ($n = 2$)
R does not adhere	.50 ($n = 10$)	1.0 ($n = 5$)

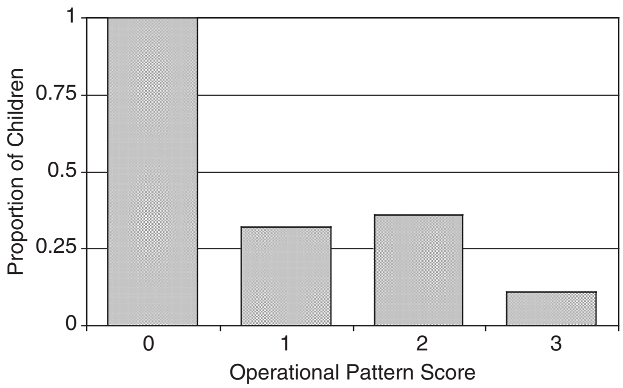

Figure 2. Proportion of children who used a correct strategy after the lesson as a function of operational pattern score.

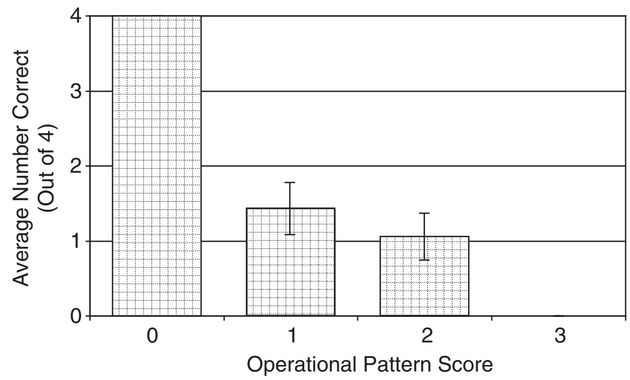

Figure 3. Average number of transfer problems solved correctly (out of 4) as a function of operational pattern score.

the other predictors in the model, $\hat{\beta} = -0.92$, $z = -2.34$, Wald $(1, N = 67) = 5.47$, $p = .02$. Figure 2 displays the relationship between operational pattern score (0–3) and the proportion of children who used a correct strategy after the brief lesson. The model estimates that the odds of using a correct strategy decrease by 2.52 for each unit increase in adherence to the operational patterns. Thus, the odds of using a correct strategy are more than 7.5 times lower for children who adhere to all three operational patterns than for children who do not adhere to any of the operational patterns. None of the other predictors was significant (problem structure lesson: $\hat{\beta} = 1.10$, $z = 1.34$; equal sign concept lesson: $\hat{\beta} = 0.15$, $z = 0.67$; interaction of lesson conditions: $\hat{\beta} = -1.34$, $z = -1.17$; age: $\hat{\beta} < .001$, $z = 0.18$). Thus, as predicted by the change-resistance account, there was a significant negative linear relationship between adherence to the operational patterns and whether children used a correct strategy after receiving a brief lesson.

Finally, we used linear regression to examine the relationship between operational pattern score (0–3) and performance on the transfer problems. The predictor variables in the model were identical to those used in the previous logistic regression models. The dependent variable was number of correctly solved transfer problems (out of 4). As shown in Figure 3, there was a significant negative linear relationship between operational pattern score and transfer performance when controlling for the other predictors in the model, $\hat{\beta} = -0.94$, $z = -3.56$, $p = .001$. The effect of problem structure lesson was marginal, $\hat{\beta} = 0.98$, $z = 1.74$, $p = .09$, and none of the other predictors was significant (equal sign concept lesson: $\hat{\beta} = 0.90$, $z = 1.63$; interaction of lesson conditions: $\hat{\beta} = -0.86$, $z = -1.06$; age: $\hat{\beta} = .0004$, $z = 0.4$). Thus, consistent with the previous analyses, adherence to the operational patterns was negatively associated with performance on equations after a brief lesson.

Do These Effects Hold When Controlling for Strategy Variability?

It is, of course, possible that the relations described thus far depend not on entrenchment of the operational patterns per se, but on strategy variability, which may be confounded with entrenchment. Many past studies have documented relations between strategy variability and learning; therefore, it is important to examine whether the relations between adherence to the operational patterns and learning hold even when controlling for strategy variability.

Only 7% of children (5 of 67) displayed variability in their problem-solving strategies on the experimental pretest; therefore, it was not possible to examine directly the relation between adherence to the operational patterns and strategy variability in this sample. One possible reason for the low level of variability observed in this experiment is that the equation-solving measure was limited to equations with the blank in final position (e.g., 3+4+5 = 3+__). McNeil and Alibali (2004) showed that equations of this type are particularly difficult for children. Another reason may be the task instructions (i.e., ''Try to solve the problem as best as you can, and then put your answer in the blank''). The instruction to put an ''answer'' may have led children to use the add-all strategy.

Although we could not examine relations between adherence to the operational patterns and strategy variability, it was possible to examine whether the relations between adherence to the operational patterns and learning held in the subsample of children who did not display strategy variability (i.e., children who used the same strategy to solve all three pretest problems, $n = 62$). Indeed, for this subsample, there was a significant negative linear relationship between operational pattern score (0–3) and strategy generation (generate or not) when controlling for the other predictors in the model, $\hat{\beta} = -1.06$, $z = -2.35$, Wald $(1, N = 62) = 5.52$, $p = .02$. Additionally, there was a significant negative linear relationship between operational pattern score and use of a correct strategy (correct or not) when controlling for the other predictors in the model, $\hat{\beta} = -0.84$, $z = -1.94$, Wald $(1, N = 62) = 3.75$, $p = .05$. Finally, there was a significant negative linear relationship between operational pattern score and transfer performance, ($\hat{\beta} = -0.87$, $z = -3.05$, $p = .004$). Thus, the findings from the subsample were consistent with those from the full sample.

Summary

The results of Experiment 1 support the change-resistance account. As children's adherence to the operational patterns increased, the odds of learning from a brief lesson on equations decreased. Children who did not adhere to any of the operational patterns were most likely to generate a correct strategy for solving the equations after a lesson. Findings corroborate the predicted relationship between children's knowledge of arithmetic operations and equation-learning difficulties. However, Experiment 1 used a correlational design. Individual differences before a lesson on equations predicted who would and would not learn from the lesson. It would be more convincing to demonstrate that knowledge of the operational patterns per se causes difficulties with equations. Thus, in Experiment 2, we activated participants' knowledge of the operational patterns and examined effects on equation-solving performance.

Experiment 2

The change-resistance account predicts that equation-solving performance should be worse when knowledge of the operational patterns is activated. To test this prediction, we compared the equation-solving performance of participants whose knowledge of operational patterns was activated with participants whose knowledge of operational patterns was not activated. We believed it would be imprudent to activate (and consequently strengthen) elementary school children's knowledge of the operational patterns because they have not yet learned to solve equations correctly with operations on both sides of the equal sign. Therefore, in Experiment 2 we chose to work with undergraduate participants. Because undergraduates already know how to solve the equations, any effects of knowledge activation would likely be ephemeral. Given the obvious differences between undergraduates and elementary school children, we acknowledge that we cannot be certain that the same processes are at work in the two groups. However, evidence for a causal link between knowledge of the operational patterns and difficulties with equations in undergraduates would help build a case for the change-resistance account. Prevailing theories of problem solving (Anderson, 2002; Anderson et al., 1996; Haverty, 1999; Kotovsky et al., 1985)—as well as traditional educational practices—predict that equation-solving performance should be facilitated when knowledge of arithmetic operations is activated, but the change-resistance account predicts the opposite.

Method

Participants

Twenty-eight undergraduates participated; 4 were excluded because of experimenter error. The final sample consisted of 24 undergraduates (8 men, 16 women; 1 African American, 21 Caucasian, 2 Hispanic) from the University of Wisconsin–Madison Introductory Psychology participant pool. Participants received one extra credit point toward their Introduction to Psychology grade for their participation.

The mathematics experience of undergraduates in the participant pool varies widely. However, the minimum mathematics experience required for admission to the University is one algebra course, one geometry course, and one advanced math course (e.g., advanced algebra, trigonometry, calculus). The average math ACT score reported by undergraduates who have participated in similar experiments in our lab is 27 (approximately the 89th percentile; ACT Inc., 2005).

Apparatus

Stimuli were presented on iMac G4 computers (17-in. screen, standard keyboard) using the PsyScope 1.2.5 interactive graphic system for experimental design and control (Cohen, MacWhinney, Flatt, & Provost, 1993).

Procedure

Participants were seated at computers in individual cubicles. The session had two main phases: an activation phase and an equation-solving phase. The purpose of the activation phase was to activate participants' knowledge of the operational patterns experimentally. Consequently, in contrast to Experiment 1, we had control over whether participants' knowledge of the operational patterns was activated. Participants were randomly assigned either to an experimental condition, in which their knowledge of all three operational patterns was activated, or to a control condition, in which their knowledge of the operational patterns was not activated. The conditions were designed to parallel the extreme operational pattern categories from Experiment 1 (i.e., adherence to the operational patterns on 3 measures and adherence to the operational patterns on 0 measures).

Activation Phase

During the activation phase, participants were presented with 24 trials in which they were shown a target followed by a set of five stimuli. For each stimulus, participants' goal was to indicate whether it matched some aspect of the target by pressing one of two keys labeled *YES* and *NO* on the keyboard. Each stimulus appeared on the screen until participants made their decision. In both the experimental and control conditions, there were 8 trials of perceptual pattern activation, 8 trials of concept activation, and 8 trials of strategy activation, each of which is described here.

Perceptual pattern activation. To activate knowledge of the "operations = answer" perceptual pattern, participants in the experimental condition were presented with a target equation (e.g., $375+659=\_\_$) followed by five stimulus equations randomly selected from a set of equations (e.g., $375+659=\_\_$, $98+673=\_\_$, $12+29+17=\_\_$, $8+7+15+9=\_\_$). Participants' goal was to decide whether each of the five stimulus equations matched the target equation. Both the target equation and the stimulus equations adhered to the traditional "operations = answer" problem structure. Participants in the control condition were presented with nonsense letter patterns (e.g., XxxX, Cccc, and RrXc) instead of equations.

Concept activation. To activate knowledge of the operational concept of addition, participants in the experimental condition were presented with a target word (e.g., Total) followed by five stimulus words randomly selected from a set of words (e.g., Total, Plus, Sum, Add). Participants' goal was to decide whether each of the five stimulus words matched the target word. Both the target word and the stimulus words were chosen to activate knowledge of the operation of addition. Participants in the control condition were presented with neutral words (e.g., Party, Tea, House) instead of words designed to activate knowledge of the operation of addition.

Strategy activation. To activate knowledge of the operational strategy of adding numbers together, participants in the experimental condition were presented with a target number (e.g., 16) followed by five addend pairs randomly selected from a set of addend pairs (e.g., 8 and 8, 10 and 5, 2 and 6, 20 and 20). Participants' goal was to decide whether each of the addend pairs summed to the target number. This task was designed to activate the strategy of adding numbers together. Participants in the control condition were presented with colors as targets (e.g., orange) and color pairs as stimulus pairs (e.g., red and yellow, red and white) instead of numbers and addend pairs. Participants' goal was to decide whether the color pairs, when mixed together, yielded the target color.

Equation-Solving Phase

Immediately following the activation phase, there was an equation-solving phase in which participants solved two typical addition problems (e.g., $5+7+3+5=\_\_$) followed by eight equations with operations on both sides of the equal sign (e.g., $7+4+5=7+\_\_$). Before each equation was presented, participants' gaze was directed to the center of the screen at the location where the equation would be presented. The equations were displayed in the center of the computer screen for a brief period ($M = 1500$ ms). Each equation was approximately 5 mm tall and 50 mm wide. After each equation was presented, participants were instructed to write the solution on a provided answer sheet. After recording each answer, participants pressed a key on the keyboard to move on to the next equation.

Coding

Participants' equation-solving strategies were coded using the system developed by Perry et al. (1988). Strategies were assigned based on participants' solutions. Solutions were coded as reflecting a particular strategy as long as they were within ± 1 of the solution that would result from that strategy. We were interested in participants' use of correct strategies and in their use of the add-all strategy. Recall that the add-all strategy is commonly used by elementary school children who have not received instruction on equations or on the concept of mathematical equivalence.

Results and Discussion

Participants' performance on the two typical addition problems (e.g., 5+7+3+5 = __) was good and did not differ between conditions. Two participants (one in each condition) did not use a correct strategy on one of the two typical addition problems; all others used a correct strategy on both typical addition problems.

Figure 4 presents a frequency plot of the number of correct strategies used on the equations with operations on both sides of the equal sign (out of 8). Specifically, the figure displays the number of participants who used 0 correct strategies, 1 correct strategy, 2 correct strategies, and so on, up to 8 correct strategies. Performance on the equations was good overall and not normally distributed, with 67% of the participants using a correct strategy on either seven ($n = 6$) or all eight ($n = 10$) equations. Given the potential ceiling effects, a parametric analysis was unwarranted; therefore, we categorized participants into two groups based on the natural, balanced separation in the data—those who used a correct strategy on at least seven of eight equations ($n = 16$) and those who used a correct strategy on fewer than seven equations ($n = 8$). We then used a nonparametric analysis to examine the effects of operational pattern activation.

As predicted, participants whose knowledge of the operational patterns was activated were less likely than participants whose knowledge was not activated to use a correct strategy on at least seven of eight equations, $\chi^2(1, N = 24) = 6.75$, $p = .009$. Of the 12 participants in the control condition, 11 (92%) used a correct strategy on at least seven of the eight equations. Of the 12 participants in the experimental condition, only 5 (42%) used a correct strategy on at least seven of the eight equations. This result is consistent with the results of Experiment 1 and supports the predictions of the change-resistance account. When people's knowledge of the operational patterns is activated, they are more likely to have difficulties with equations. The result challenges theories of mathematics learning that hold that knowledge of arithmetic operations should facilitate performance on more complex equations. It also challenges the intuitive hypothesis that people who are primed with mathematical tasks will perform better on subsequent math problems than people who are primed with nonmathematical tasks. Participants in the control group performed nonmathematical tasks and subsequently performed better than participants in the experimental group on mathematical equations.

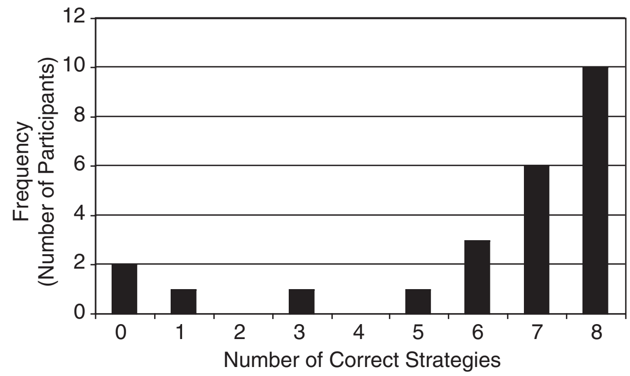

Figure 4. Frequency plot of the number of correct strategies used on the equations with operations on both sides of the equal sign (out of 8).

The change-resistance account predicts that equation-solving performance suffers when knowledge of the operational patterns is activated because knowledge of the operational patterns controls attention, determines what gets encoded, and influences how information is interpreted. Thus, when participants' knowledge of the operational patterns is activated, they should be more likely to view equations operationally and adhere to the operational, add-all strategy (e.g., 7+4+5 = 7+<u>23</u>) in solving equations. Indeed, whether participants ever used the add-all strategy was contingent on condition, $\chi^2(1, N = 24) = 6.32$, $p = .01$. None of the 12 participants in the control condition ever used the add-all strategy, not even on one equation, whereas 5 of 12 participants (42%) in the experimental condition used the add-all strategy at least once. In fact, all 5 participants in the experimental condition who used the add-all strategy used it more than once ($M = 3.8$ of 8 equations). Use of the add-all strategy accounted for 49% of the incorrect strategies used by participants in the experimental condition. Thus, when undergraduates' knowledge of the operational patterns is activated, they tend to resemble elementary school children and solve equations by adding all the numbers.

The results of Experiment 2 provide additional support for the change-resistance account. Participants who had their knowledge of the three operational patterns activated did not perform well on equations, and they were likely to err by adding all the numbers. In contrast, participants who did not have any of the three operational patterns activated performed well on equations and were unlikely to add all the numbers. Although these results are consistent with the change-resistance account, two caveats should be noted when making direct comparisons between Experiment 1 and Experiment 2. First, elementary school children and undergraduates differ on several dimensions, including age and experience with math. Second, there were procedural differences between the two experiments, including method of presenting the equations (easel vs. computer) and amount of time each equation was presented (no time restriction vs. brief presentation). Despite these limitations, however, results of Experiment 2 help build the case for the change-resistance account by establishing a casual link between activation of the operational patterns and difficulties solving equations.

General Discussion

The experiments presented in this article tested whether knowledge of arithmetic operations contributes to difficulties with equations. In Experiment 1, children's adherence to three operational patterns that commonly occur in arithmetic was negatively associated with the likelihood of learning from a brief lesson on equations. In Experiment 2, participants whose knowledge of the three operational patterns was activated were less likely than participants whose knowledge was not activated to perform well on equations. These findings support the change-resistance account and suggest that knowledge of arithmetic operations hinders performance on and learning of equations. We believe that these findings have important theoretical and educational implications, as discussed next. However, it is important to note that it would be premature to generalize the results to classroom learning, given that many aspects of our experiments were far removed from typical classroom settings.

The change-resistance account focuses on what children have, and thus, places substantial weight on the current state of the cognitive system. This differs from accounts of children's learning difficulties that focus on what children lack. Accounts that focus on what children lack generally assume that the cognitive system is moving, steadily or in spurts, toward some common adult state (Kessen, 1984). Each period of immature development is seen as the adult state minus something. In turn, the research strategy has been to start with what is known about the adult state and work backward in hopes of discovering mechanisms that give rise to the adult state. This strategy may be effective for understanding well-defined problems in the laboratory; however, cognitive development is not a well-defined problem. Different processes can lead to the same end state, and the same processes can lead to different end states. Thus, although end states provide an inventory of potentials for development, they do little to inform our understanding of the processes that create them.

The change-resistance account argues that to understand learning and cognitive development, it may not be necessary or even desirable to specify an end state. Instead, development can be viewed as a succession of attempts to preserve the current state in the face of an external impetus for change. As Kuo (1976) eloquently argued, the study of development is not about endpoints, but potentials. Scientists who start with the current state in mind can hypothesize about potential subsequent states and experimentally examine conditions under which desired future states can be achieved from the current state. The goal of this research strategy, then, is to uncover the mechanisms that drive the cognitive system—both mechanisms of change and mechanisms of change resistance.

One mechanism of change resistance in children's mathematical development may be entrenched knowledge of operational patterns. We suggest that knowledge of operational patterns should be most entrenched between third and fifth grades. Before third grade, children are just starting to learn about arithmetic operations; therefore, their knowledge of operational patterns is weak. Between third and fifth grades, children continue practicing arithmetic procedures, and their knowledge of operational patterns becomes stronger and more robust. As a result, they inflexibly apply their knowledge of the operational patterns and are unable to invent new strategies (i.e., resist change) when presented with novel equations (cf. Hatano, 1988). This leads to the unintuitive prediction, which we are currently testing, that first- and second-grade children may be more likely than third- through fifth-grade children to solve equations with operations on both sides correctly. After fifth grade, children are introduced to prealgebra; therefore, the operational patterns begin to lose their predictive power, and children's representations of these patterns decrease in strength.

The present results contribute to the growing body of evidence suggesting that prior knowledge can be detrimental in some situations (Adelson, 1984; Bruner & Postman, 1949; Flege, Frieda, & Nozawa, 1997; Gray & Fu, 2001; Knoblich et al., 1999; Kuhl, 2000; Wiley, 1998). These findings present a paradox because knowledge is often cited as something that facilitates learning (Rittle-Johnson, Siegler, & Alibali, 2001), memory (Chase & Simon, 1973; Chi, 1978), and problem solving (Larkin, 1983). Indeed, familiar proverbs such as ''knowledge is power'' highlight that people intuitively think knowledge is a good thing. Because knowledge is typically beneficial, situations in which knowledge hinders performance—like the those described in this article—provide a unique window into how the cognitive system works.

The present study also extends past research about strategy variability as a predictor of cognitive change. The results shed light on why some individuals with low strategy variability nevertheless change—namely, because their existing strategies are not entrenched. People may use an incorrect strategy consistently, not because it is entrenched but because they cannot think of any other way to solve the problem. They may know that their existing strategy is incorrect, but they may not have the knowledge necessary to construct a new strategy. Unlike people who are entrenched in a particular strategy, these individuals may be highly receptive to instruction. When they encounter new information, they may readily incorporate that information into their knowledge base and abandon their incorrect strategy in favor of a new strategy. More broadly, the present work highlights the need for research to address directly the relations among variability, entrenchment, and learning.

Perhaps the most surprising finding of the present research was that undergraduates could be made to perform like elementary school children. When undergraduates' knowledge of the operational patterns was activated in Experiment 2, some started solving fairly straightforward equations (e.g., $7+4+5=7+\_\_$) by adding all the numbers. This strategy is often used by children who have not yet received instruction on such equations (McNeil & Alibali, 2000). We argue that this finding is a testament to the power of early learning. Equations such as $7+4+5=7+\_\_$ would typically be trivial for college students, who have years of experience with algebraic equations. Nonetheless, after just a few minutes of exposure to the operational patterns that they learned many years previously, their performance on such equations suffered. These results are consistent with other studies that demonstrate the power of individuals' initial associations in a wide variety of domains (e.g., infants' spatial memory, Munakata, McClelland, Johnson, & Siegler, 1997; Spencer, Smith, & Thelen, 2001; children's strategies for solving science and mathematics problems, Schauble, 1990; Siegler & Stern, 1998; adults learning a second language, Best, McRoberts, & Goodell, 2001; Flege et al., 1999; nonhuman animals' latent inhibition and spontaneous recovery, Bouton, Nelson, & Rosas, 1999; Tolman, 1948). Results also converge with other studies showing that undergraduates sometimes use less advanced strategies for solving math problems than might be expected (e.g., Clement, Lochhead, & Monk, 1981; LeFevre, Smith-Chant, Hiscock, Daley, & Morris, 2003; Rosnick, 1981).

Although it is tempting to assume that the underlying processes responsible for undergraduates' difficulties in Experiment 2 are the same as the processes responsible for children's difficulties in Experiment 1, one cannot be certain that this is the case. The age- and experience-related differences between the two participant groups, as well as the procedural differences between the two experiments, discourage us from making strong claims about the similarities between the two cases. Instead, we argue that Experiment 2 should be viewed primarily as an existence proof that knowledge of the operational patterns hinders performance on equations. Taken together with the results of Experiment 1, this suggests that knowledge of arithmetic operations (as currently taught in the United States) may contribute to difficulties with learning of and performance on equations.

In practical terms, the present findings challenge some traditional educational practices. In most schools in the United States, children learn arithmetic operations for many years before reaching algebra and being formally introduced to equations and the concept of mathematical equivalence. This instructional strategy makes sense in the context of accounts that attribute children's difficulties with equations to something that children lack. The argument is as follows: If elementary school children lack the necessary domain-general logical structures or working memory resources for understanding complex equations, why should teachers waste valuable class time trying to teach something that children are not ''developmentally ready'' to learn? Instead, teachers should focus on teaching something children are capable of learning—arithmetic operations. Similarly, if children's difficulties with equations are due to their lack of proficiency with arithmetic operations, it makes sense for children to spend years practicing arithmetic operations before moving onto complex equations. However, this instructional strategy makes less sense within the context of the change-resistance account, which argues that children's knowledge of the operational patterns becomes entrenched through years of practice with arithmetic operations, and as a result, children's ability to learn about more complex equations suffers. According to this view, it would make more sense to start teaching children about complex equations as early as possible—before knowledge of the operational patterns becomes entrenched. Along these lines, many mathematics education researchers have argued for the ''algebrafication'' of elementary school mathematics (Blanton & Kaput, 2003; Bodanskii, 1991; Carpenter et al., 2003; Carraher, Schliemann, & Brizuela, 2001).

Although the present findings lead us to question some educational practices that are common in the United States, we must be cautious about our conclusions. It would be premature for us to conclude that knowledge of arithmetic operations hinders the learning of children in classroom settings. Although the results of Experiment 1 hint in that direction, the jury is still out. Indeed, our experiments do not rule out the possibility that children need even more practice with arithmetic operations before progressing to algebra. It may be that the relationship between practice with arithmetic and performance on equations is U-shaped, with an intermediate level of arithmetic proficiency being most detrimental (cf. Dowker, Flood, Griffiths, Harriss, & Hook, 1996). We argue that third- through fifth-grade children develop routine expertise with arithmetic operations, and this leads them to apply their knowledge of operational patterns inflexibly (cf. Hatano, 1988). With even more practice, however, children may eventually develop adaptive expertise and exhibit the creativity and flexibility of true experts (Dowker, 1992; Hatano, 1988). We are currently investigating the causal link between knowledge of arithmetic operations and mathematics learning difficulties in classroom settings.

We contend that change resistance extends beyond the domain of mathematics. Indeed, it may be a fundamental characteristic of the cognitive system. It is manifested in a wide variety of domains, including science (Schauble, 1990) and foreign language (Kuhl, 2000). Thus, any general theory of learning and cognitive development needs to address not only mechanisms of change but also mechanisms of change resistance.

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