Optional Handout #1

Fall 1995, Alias, "The class that asked for algebra"

Problem: Show that these two expressions for SS_{A_2} are equivalent:

(1) $SS_A = [A] - [T]$ (2) $SS_A = n\sum_{i=1}^{a}(\overline{Y}_{A_i} - \overline{Y_T})^2$

Facts:

$$\overline{Y_T} = \frac{\sum Y_{A_{ij}}}{an} = \frac{T}{an} \qquad [T] = \frac{T^2}{an}$$

$$\overline{Y_{A_i}} = \frac{\sum Y_{A_i}}{n} = \frac{A_i}{n} \qquad [A] = \frac{\sum_{i=1}^{a} A_i^{\,2}}{n}$$

Start with (2), $SS_A = n\sum_{i=1}^{a}(\overline{Y_{A_i}} - \overline{Y_T})^2$

$$= n\sum(\frac{A_i}{n} - \frac{T}{an})^2 = n\sum(\frac{A_i^2}{n^2} - 2\frac{A_iT}{an^2} + \frac{T^2}{a^2n^2})$$

$$= \sum(\frac{A_i^2}{n} - 2\frac{A_iT}{an} + \frac{T}{a^2n}) =$$

$$= \frac{\sum A_i^2}{n} - 2\sum\frac{A_iT}{an} + \sum\frac{T^2}{a^2n}$$

Now we recognize [A].

$$= [A] - 2\sum\frac{A_iT}{an} = \sum\frac{T^2}{a^2n}$$

Notice that T is a constant. When we sum over T, we get a t, or we can put ,T in front of the sum sign.

$$= [A] - \frac{2T}{an}\sum A_i + \frac{aT^2}{a^2n}$$

Now we recognize that $\sum A_i = T$.

$$= [A] - \frac{2T^2}{an} + \frac{T^2}{an} = [A] - \frac{T^2}{an}$$

Now we recognize $[T] = \frac{T^2}{an}$.

$$= [A] - [T]$$

$SS_{S/A}$

$$SS_{S/A} = \sum_{groups} [\sum_{j} (Y_{ij} - \overline{Y_{A_i}})^2]$$

Square out inside sum: $= \sum_{groups} [\sum_{j} \{Y_{ij}^2 - 2\overline{Y_{A_i}} Y_{ij} + \overline{Y_{A_i}^2}\}]$

Distribute inside sum sign:

$$= \sum_{groups} [\sum_{j} Y_{ij}^2 - 2\overline{Y_{A_i}} \sum_{j} Y_{ij} + \sum_{j} \overline{Y_{A_i}^2}]$$

Distribute outside sum sign:

$$= \sum_{groups} \sum_{j} Y_{ij}^2 - \sum_{groups} (2\overline{Y_{A_i}} \sum_{j} Y_{ij}) + \sum_{groups} (\sum_{j} \overline{Y_{A_i}^2})$$

Replace $\sum\sum Y_{i_j}^2$ with $[Y]$; Replace $\overline{Y_{A_i}}$ with $\frac{A_i}{n}$:

$$= [Y] - \sum_{groups} (2\frac{A_i}{n} \sum_{j} Y_{ij}) - \sum_{groups} (\sum_{j} \frac{A_i^2}{n^2}).$$

Replace $\sum_{j} Y_{ij}$ with A_{ij} replace $\sum_{j} (\frac{A_i}{n})$ with $(n)(\frac{A_i^2}{n^2})$:

$$= [Y] - \sum_{groups} (2\frac{A_i}{n} . A_i) - \sum_{groups} (n\frac{A_i^2}{n^2})$$

$$= [Y] - 2\sum_{groups} (\frac{A_i^2}{n}) - \sum_{groups} (\frac{A_i^2}{n})$$

$= [Y] - \sum_{groups} (\frac{A_i^2}{n})$ Replace $\sum_{groups} (\frac{A_i^2}{n})$ with $[A]$:

$SS_{S/A} = [Y] - [A]$.

(1999)