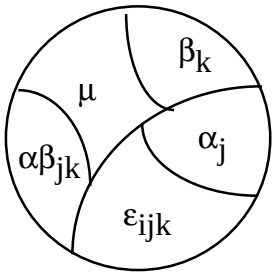
Psychology 610 Handout #14
Prof. Moore

GENERAL LINEAR --Two Way ANOVA

Score model:

Let Y_{ijk} be the i^{th} subject in cell AB_{jk}

Y_{ijk}



$$Y_{ijk} = \bar{Y}_T + (\bar{Y}_j - \bar{Y}_T) + (\bar{Y}_k - \bar{Y}_T) + (\bar{Y}_{jk} - \bar{Y}_j - \bar{Y}_k + \bar{Y}_T) + (Y_{ijk} - \bar{Y}_{jk})$$

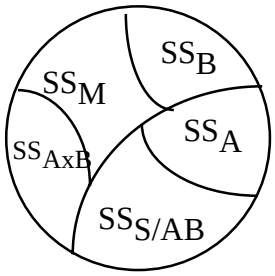
$$Y_{ijk} = \mu + \alpha_j + \beta_k + \alpha\beta_{jk} + \varepsilon_{ijk}$$

$$Y_{ijk} = \text{mean} + \underset{\text{effect}}{A} + \underset{\text{effect}}{B} + \underset{\text{interaction effect}}{A \times B} + \text{error}$$

(Note: $\alpha\beta_{jk} = \bar{Y}_{jk} - \alpha_j - \beta_k - \mu$, what is systematic in the cell mean after lower order effects are subtracted from it.)

SSS

BREAKDOWN OF GRAND TOTAL OF SUM OF SQUARED SCORES

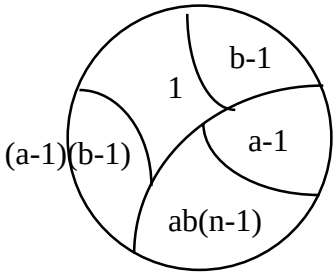


$$SSS = \Sigma Y_{ijk}^2 = SS_{mean} + SS_A + SS_B + SS_{AxB} + SS_{S/AB}$$

BREAKDOWN OF DEGREES OF FREEDOM

total observations = abn

$$abn = \underset{\text{df mean}}{1} + \underset{\text{df for A}}{(a - 1)} + \underset{\text{df for B}}{(b - 1)} + \underset{\text{df for A x B}}{(a - 1)(b - 1)} + \underset{\text{df for S/AB}}{a(n - 1)}$$



TWO-WAY FIXED FACTOR BETWEEN-S DESIGNS
Expected Values of the Mean Squares

$E[SS_M] = \sigma_e^2 + abn\mu^2$

$E[SS_A] = (a-1)(\sigma_e^2 + nb\theta_A^2)$ $\quad E[SS_{AB}] = (a-1)(b-1)(\sigma_e^2 + n\theta_{AB}^2)$

$E[SS_B] = (b-1)(\sigma_e^2 + na\theta_B^2)$

$E[SS_{S/AB}] = ab(n-1)\sigma_e^2$

In general: MS = SS/df

$E[MS_M] = \sigma_e^2 + abn\mu^2$ $\quad E[MS_B] = \sigma_e^2 + na\theta_B^2$

$E[MS_A] = \sigma_e^2 + nb\theta_A^2$ $\quad E[MS_{AB}] = \sigma_e^2 + n\theta_{AB}^2$

$E[MS_{S/AB}] = \sigma_e^2$

Note: θ^2 is used for the variance of fixed factors

σ^2 is used for the variance of random factors; subjects is a random factor.

$F_A = MS_A/MS_{S/A}$ $\quad E[F[H_0 true] = df_{error} / df_{error}^{-2} = \dfrac{a(n-1)}{a(n-1)-2}$

~ 1 as df_{error} get large