1. Example from Kirk, pp. 353 ff. Latin Square "Crossover Design"

	A						S Totals	$\bar{Y}_{PA}$
	A_1		A_2		A_3			
S_1	B_1	7	B_2	14	B_3	12	33	11.0
S_2	B_1	3	B_3	11	B_2	5	19	6.33
S_3	B_2	7	B_3	11	B_1	6	24	8.0
S_4	B_3	9	B_1	12	B_2	13	34	11.33
S_5	B_2	9	B_1	7	B_3	8	24	8.0
S_6	B_3	9	B_2	13	B_1	8	30	10.0
Totals A =		44		68		52	T=164	$\bar{Y}_T$=9.11
$\bar{Y}_{Aj}$		7.33		11.33		8.67		

$[Y] = \sum Y^2 = 1652$

$[T] = 164^2/18 = 1494.22$

$[S] = 4658/3 = 1552.67$

$[A] = 9264/6 = 1544$

$[B] = 9170/6 = 1528.33$

	B_1	B_2	B_3
	7	7	9
	3	9	9
	12	14	11
	7	13	11
	6	5	12
	8	13	8
B =	43	61	60
$\bar{Y}_{Bk}$	7.17	10.17	10.00

ANOVA Table

Source	df	SS	MS	F	
Mean	1	[T] = 1494.22	1494.22	773.99	
A	a-1 = 2	[A] – [T] = 49.78	24.89	12.89	p < .01
B	b-1 = 2	[B] – [T] = 34.11	17.06	8.83	p < .01
Subjects	n-1 = 5	[S] – [T] = 58.44	11.69		
Error	(ℓ-1)(n-2) = 8 or "residual df"	= 15.44 [Y] - [A] - [B] - [S] + 2[T], or "residual"	1.93		
Total	Total obs = 18 ℓn = (3)(6)	[Y] = 1652			

Let ℓ = # levels in the Latin Square
In a Latin Square, a = b = ℓ

Linear model:

<u>Population</u>: $Y_{ijk} = \mu + \alpha_j + \beta_k + \pi_i + \varepsilon_{ijk}$

where $\alpha_j = \mu_j - \mu$ and $\sum\alpha_j = 0$

$\beta_k = \mu_k - \mu$ and $\sum\beta_k = 0$

$\pi_i = \mu_i - \mu$ and $\sim N(0, \sigma_\pi^2)$

$\varepsilon_{ijk} = Y_{ijk} - \mu - \alpha_j - \beta_k - \pi_i$ $\varepsilon_{ijk} \sim N(0, \sigma_e^2)$

<u>Sample</u>: $Y_{ijk} = \overline{Y_T} + (\overline{Y_{Aj}} - \overline{Y_T}) + (\overline{Y_{B_k}} - \overline{Y_T}) + (\overline{Y_{P_i}} - \overline{Y_T}) + \varepsilon_{ijk}$

$\hat{\alpha}_j = \overline{Y_{Aj}} - \overline{Y_T}$ $\hat{\beta}_k = \overline{Y_{B_k}} - \overline{Y_T}$

$\varepsilon_{ijk} = Y_{ijk} - \overline{Y_{A_j}} - \overline{Y_{B_k}} - \overline{Y_{P_i}} + 2\overline{Y_T}$