Nested Designs
With and Without Random Factors

I. Two-way design

Factor B is nested in A if the levels of B are not comparable across the levels of A. For this design B is nested in A, and participants are nested in treatment cells.

	A_1						A_2						
	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	B11	B12	
	6	2	5	4	6	1	8	1	9	7	9	5	
	8	3	6	3	7	3	6	3	7	4	6	5	
	4	5	3	2	8	5	7	3	8	6	8	7	
AB_{jk}	18	10	14	9	21	9	21	7	24	17	23	17	T=190

$A_1 = 81$ $A_2 = 109$

$\bar{Y}_{A1} = 4.50$ $\bar{Y}_{A2} = 6.06$

$[Y] = 437$

Linear model for nested design:

$$Y_{ijk} = \mu + \alpha_j + \beta_{k(j)} + \varepsilon_{ijk}, \text{ where } \alpha_j = \mu_j - \mu,\ \beta_{k(j)} = \mu_{jk} - \mu_j$$

Side conditions for a fixed factor: $\Sigma\alpha_j = 0$

Side conditions for a random factor: $\beta_{k(j)} \sim N(0, \sigma^2_{B/A})$

B assumed <u>random</u>. A is fixed.

Source	df	SS	MS	F
Mean	1	1002.78	1002.78	93.37
A	a - 1 = 1	21.78	21.78	2.03
B/A	a(b - 1) = 10	107.45	10.74	5.37*
S/AB	ab(n - 1) = 24	48.00	2.00	

*p< .01

-- Relate to simple main effects!
-- Compare analyses with different designs and assumptions!

Expected mean squares

	B random and nested	B fixed and nested
Mean	$abn\mu^2 + n\sigma^2_{B/A} + \sigma^2_e$	$abn\mu^2 + \sigma^2_e$
A	$bn\theta^2_A + n\sigma^2_{B/A} + \sigma^2_e$	$bn\theta^2_A + \sigma^2_e$
B/A	$n\sigma^2_{B/A} + \sigma_e{}^2$	$n\theta^2_{B/A} + \sigma^2_e$
S/AB	σ^2_e	σ^2_e

A x B factorial (crossed) design

	B fixed	B random
Mean	$abn\mu^2 + \sigma^2_e$	$abn\mu^2 + n\sigma^2_{AB} + \sigma^2_e$
A	$bn\theta^2_A + \sigma\gamma_e$	$bn\theta_A{}^2 + n\sigma^2_{AB} + \sigma^2_e$
B	$an\theta^2_B + \sigma^2_e$	$an\sigma^2_B + \sigma^2_e$
A x B	$n\theta^2_{AB} + \sigma^2_e$	$n\sigma^2_{AB} + \sigma^2_e$
S/AB	σ^2_e	σ^2_e

II. A 3-way nested design

Linear model: $Y_{ijkl} = \mu + \alpha_j + \beta_k + \gamma_{l(k)} + \alpha\beta_{jk} + \alpha\gamma_{jl(k)} + \varepsilon_{ijkl}$,
Side conditions: $\Sigma\alpha_j = 0$, $\Sigma\beta_j = 0$ (factors A and B fixed), $\gamma_{l(k)} \sim N(0, \sigma_\gamma^2)$ (factor C random)

where $\alpha_j = \mu_j - \mu$, $\beta_k = \mu_k - \mu$, $\gamma_{l(k)} = \mu_{kl} - \mu_k$, etc.

			Sentence Stem								
		C	Person			Man			Woman		
B (Role Gender)	Male	Ex1	6	8	4	2	3	5	10	11	7
		Ex2	5	6	3	4	3	2	9	8	9
		Ex3	6	7	5	1	3	5	7	11	9
		Ex4	8	5	7	5	6	3	7	6	10
	Female	Ex5	8	9	7	11	12	8	3	4	6
		Ex6	6	7	4	8	9	11	5	4	3
		Ex7	7	8	6	8	12	10	2	3	6
		Ex8	9	6	8	8	7	11	5	7	4

<u>Source</u>	<u>df</u>	<u>SS</u>	<u>MS</u>	<u>F</u>	<u>Error</u>
Mean	1	3042.0	3042.0	1579.44	C/B
A(Stem)	a - 1 = 2	.083	.042	.021	AxC/B
B(Stem)	b - 1 = 1	18.0	18.0	9.346	C/B
A x B	(a-1)(b-1) = 2	326.083	163.042	80.59	AxC/B
C/B	b(c - 1) = 6	11.556	1.926	.72	S/ABC
C/BxA	b(c-1)(a-1) = 12	24.278	2.023	.759	S/ABC
S/ABC	abc(n - 1) = 48	128.0	2.667		

Expected Mean Squares - Assuming C random, A and B fixed

Mean	$abcn\mu^2 + an\sigma^2_{C/B} + \sigma_e^2$
A	$bcn\theta^2_A + n\sigma^2_{C/BxA} + \sigma_e^2$
B	$acn\theta^2_B + an\sigma^2_{C/B} + \sigma_e^2$
A x B	$cn\theta^2_{AxB} + n\sigma^2_{C/BxA} + \sigma_e^2$
C/B	$an\sigma^2_{C/B} + \sigma_e^2$
C/BxA	$n\sigma^2_{C/BxA} + \sigma_e^2$
S/ABC	σ_e^2

General Principle:

- -- When a random factor is crossed with a fixed factor, the EMS for the fixed factor includes the interaction of the random factor with itself, and so that interaction becomes the error term for testing the fixed factor.
- -- When a random factor is nested in a fixed factor, the EMS for the fixed factor includes the nested factor, and so the nested factor becomes the error term for testing the fixed factor.