

RELATIONSHIPS BETWEEN ONE-WAY AND TWO-WAY DESIGNS

1. The data below represent the number of errors made by subjects in a paired-associate learning task. There are four groups, with eight subjects per group. The experimenter is interested in the influence of two specific but unnamed drugs on performance. Group A₁ is a control group not given any drug. Groups A₂, A₃, and A₄ are experimental groups; Group A₂ was given one drug, Group A₃ another, and Group A₄ was given both drugs.

	A ₁	A ₂	A ₃	A ₄	
	7	13	10	14	
	1	12	12	13	
	8	6	4	14	
	9	10	11	14	
	9	13	7	17	
	7	13	8	11	
	4	6	12	13	
	9	10	5	14	
Total	54	83	69	110	316
Means	6.75	10.38	8.63	13.75	9.88
$\sum Y^2$	422	923	663	1532	
$s^2 =$	8.2143	8.8393	9.6964	2.7857	

2. Partition of SS for Data

$$\begin{aligned} \sum Y^2 &= 3540 \\ SS_{\text{mean}} &= 3120.50 \\ SS_{\text{cells}} &= 212.75 \\ SS_{S/\text{cells}} &= 206.75 \end{aligned} \quad MS_{S/AB} = 28 = 7.3839$$

3. Analysis of the Data as a Two-way Design

		Drug 1		
		Absent	Present	
Drug 2	Absent	A ₁ 54	A ₂ 83	137
	Present	A ₃ 69	A ₄ 110	179
		123	193	316

Entries are totals

$$[T] = 316^2/32 = 3120.50$$

$$[Y] = 3540$$

$$[C] = (123^2 + 193^2)/16 = 3273.63$$

$$[R] = (137^2 + 179^2)/16 = 3175.63$$

$$[RC] = (54^2 + 83^2 + 69^2 + 110^2)/8 = 3333.25$$

$$SS_M = [T] = 3120.50$$

$$SS_C = [C] - [T] = 3273.63 - 3120.50 = 153.13$$

$$SS_R = [R] - [T] = 55.13$$

$$SS_{RC} = [RC] - [C] - [R] + [T] = 3333.25 - 3273.63 - 3175.63 + 3120.50 = 4.49$$

$$SS_{S/AB} = [Y] - [RC] = 3540 - 3333.25 = 206.75$$

Note that SS_{cells} from homework problem (212.75) is equal to the sum of

SS_C	153.13
SS_R	55.13
SS_{RC}	<u>4.49</u>
	212.75

4. Analysis of the Data as a Set of Orthogonal Contrasts

	A ₁	A ₂	A ₃	A ₄	
C ₁	-1	-1	+1	+1	Main effect of Drug 2
C ₂	+1	-1	+1	-1	Main effect of Drug 1
C ₃	-1	+1	+1	-1	Interaction

$$C_1 = \frac{8[-6.75 - 10.38 + 8.63 + 13.75]^2}{4} = 55.13 = SS_{\text{ROW}}$$

$$C_2 = \frac{8[6.75 - 10.38 + 8.63 - 13.75]^2}{4} = 153.13 = SS_{\text{COL}}$$

$$C_3 = \frac{8[-6.75 + 10.38 + 8.63 - 13.75]^2}{4} = 4.44 = SS_{\text{RC}}$$

Total 212.70 = SS_{CELLS} (within rounding error)