PARTIAL INTERACTIONS

(No longer covered due to time constraints;
Handout included for your future reference)

		AROUSAL				
		L	ML	MH	H	$\bar{Y}_{P_k}$
	H	3	4	2	2	2.75
PROBLEM	M	2	5	5	2	3.50
	E	2	3	5	4	3.50
	$\bar{Y}_{A_j}$	2.33	4.0	4.0	2.67	

$MS_{S/AP} = 2.30$
$df_{S/AP} = 48$
$n = 5$

Subjects perform easy, medium, or hard <u>problems</u> at one of four levels of <u>arousal</u>: low, medium low, medium high, and high. The prediction is that each problem will produce quadratic data, but the peaks will not necessarily be at the same place.

QUADRATIC COEFFS

	L	ML	MH	H	Tot
H	-1	1	1	-1	0
M	-1	1	1	-1	0
E	-1	1	1	-1	0
Tot	-3	3	3	-3	

We will test using the partial interactions
Arousal_Q x Problem

Start by doing separate SS_Q for each problem.

$$SS_{Q@HARD} = \frac{5(-3+4+2-2)}{4} = 1.25$$

$$SS_{Q@MEDIUM} = \frac{5(-2+5+5-2)}{4} = 45$$

$$SS_{Q@EASY} = \frac{5(-2+3+5-4)}{4} = 5$$

The sum of these three SS values indicates how much overall "quadraticness" there is in the data set as a function of arousal level. But it overestimates the $SS_{AROUSALQ \times PROBLEM}$ because it includes the part that is reflected in the marginal means for arousal. Note that the row quadratic coefficient sums equal zero but the column coefficient sums have the quadratic pattern. Therefore we need to correct for this overestimate by calculating and subtracting out the $SS_{AROUSAL\ QUAD}$.

$$SS_Q = \frac{15(-2.33+4+4-2.67)^2}{4} = 33.75$$

$$SS_{A\ QUAD \times P} = \sum_{k=1}^{p} SS_{A\ QUAD@P_k} - SS_{A\ QUAD}$$

$= 51.25 - 33.75$

$= 17.5$

To test this, we must know how many df we have in the $SS_{A\ QUAD\ x\ P}$. There are two ways to think about this.

Method 1: The formula for the df should reflect the formula for the SS. Therefore

$$df_{A\ QUAD\ x\ P} = \sum df_{A\ QUAD@P_k} - df_{A\ QUAD}$$

$$= p \times 1 - 1$$

$$= 2$$

Method 2: The df for an interaction always equals the product of the individual dfs.

$$Df_{A\ QUAD\ x\ P} = df_{A\ QUAD} \times df_P$$

$$= 1 \times (p-1)$$

$$= p - 1$$

$$= 2$$

So, we have $SS_{A\ QUAD\ x\ P} = 17.5$ on 2 df

$MS_{A\ QUAD\ x\ P} = 8.75$

$F_{A\ QUAD\ x\ P} = 8.75/2.30 = 3.80$

on (2, 48) df

$p < .05$

RELATION BETWEEN PARTIAL INTERACTIONS AND TWO-WAY CONTRASTS

The partial interaction idea says there will be a particular pattern on a factor (e.g., quadratic arousal factor) that will differ as a function of another factor (e.g., problem).

Two-way contrasts make us specify a particular pattern on both factors. How many such different (i.e., orthogonal) patterns could we specify on the problems factor if we wanted to? Answer: 2 (since df for Problems equals 2).

Let's pick two orthogonal contrasts arbitrarily, say $problem_{LIN}$ and $problem_{QUAD}$ and compute the two two-way contrasts.

$$SS_{A\ QUAD}\ x\ P_{LIN} = \frac{5(3-4-2+2-2+3+5-4)^2}{8}$$

Coefficients

	-1	1	1	-1
-1	1	-1	-1	1
0	0	0	0	0
1	-1	1	1	-1

$= 5(1)^2/8$

$= .625$

$$SS_{A\ QUAD}\ x\ P_{QUAD} = \frac{5(3-4-2+2-4+10+10-4+2-3-5+4)^2}{24}$$

Coefficients

	-1	1	1	-1
-1	1	-1	-1	1
2	-2	2	2	-2
-1	1	-1	-1	1

$= 5(9)^2/24$

$= 16.875$

Now we see that

$$SS_{A\ QUAD\ x\ P\ LIN} + SS_{A\ QUAD\ x\ P\ QUAD} = SS_{A\ QUAD\ x\ PROBLEM}$$

$$.625 \qquad 16.875 \qquad = \qquad 17.5$$

More generally, $SS_{A\ COMP\ x\ B}$ is the pooled SS that would be found in the subset of orthogonal 2-way contrasts that specify A_{COMP} combined with all the different B_{COMP}'s.

RELATION BETWEEN PARTIAL INTERACTIONS AND SIMPLE MAIN EFFECTS

Recall that

$$\sum SS_{A@B} = SS_A + SS_{AB}$$

We can rearrange this to

$$SS_{AB} = \sum SS_{A@B} - SS_A$$

Note the similarity to the equation for the partial interaction.

$$SS_{A\ QUAD\ x\ B} = \sum SS_{A\ QUAD@B} - SS_{A\ QUAD}$$

The logic of the two cases is identical, but our focus differs. In the case of simple main effects we want to divide our 2-way design into 1-way designs, and so we perform an operation that effectively pools the SS for a main effect and an interaction.

In the case of partial interaction, we are interested in a component of the interaction. Our operations inflate this by a component of a main effect and we must subtract that out.