

Contrasts, Comparisons and Simple Effects on Repeated Measures Designs

I. Alternate method for computing single df contrasts on repeated measures

Data	A1	A2	A3
S1	13	24	22
S2	6	30	29
S3	25	13	23
S4	20	16	25
S5	25	37	16
S6	19	30	12

A. Suppose we want to compare A₁ and A₃

1. First compute the contrast value $\psi = (-1)(\bar{Y}_{A1}) + (1)(\bar{Y}_{A3})$ for each subject.

	$\hat{\psi}$
S1	9
S2	23
S3	-2
S4	5
S5	-9
S6	-7

$$\sum \psi = 19$$

2. Compute $[\psi] = \sum \psi^2 / 1 = 769$

3. Compute $[T] = 19^2 / 6 = 60.17$

4. Compute $[\psi] - [T] = 708.83$

2. Put together as “one-group anova.” This partitions the total variance into the grand mean and error. Then we test $H_0: \mu = 0$.

Source	df	EQ	SS	MS	F
Mean	1	[T]	60.17	60.17	.42
Error	5	$[\psi] - [T]$	708.83	141.77	--

Note that this procedure gives the correct F-ratio for A_{comp} , but the SS_{Mean} (60.17) and the SS_{Error} (141.77) are on a different scale from the original analysis. If you need the actual values of the SS, you will need to divide each of these by $n^* \sum c_j^2$, where n^* is the number of observations in the score to which the coefficients have been applied. Ordinarily, this will be 1, i.e., one observation per subject per cell. However, n^* can be greater if the scores in the analysis are totals pooled over replications or pooled over some other factors.

For illustration, the normalized Ss for the present problem are:

$$SS_{A_{\text{comp}}} = SS_{\text{mean}}/2 = 30.09$$

$$SS_{SxA_{\text{comp}}} = SS_{\text{error}}/2 = 354.42$$

B. What about Keppel's more "complex" contrast, A1 & A3 versus A2 ? Just use the same procedure exactly.

1. First compute $(-1)(\bar{Y}_{A1}) + (-2)(\bar{Y}_{A2}) + (1)(\bar{Y}_{A3})$ for each subject.

	$\hat{\psi}'$
S1	-13
S2	-25
S3	22
S4	13
S5	-33
S6	<u>-29</u>

2. Compute $[\psi'] = \sum \psi'^2 / 1 = 3377$

3. Compute $[T'] = -65^2 / 6 = 704.17$

4. Compute $[\psi'] - [T'] = 2672.83$

$$\sum \psi' = -65$$

2. Put together as one-group anova and test $H_0: \mu = 0$.

Source	df	EQ	SS	MS	F
Mean	1	$[T']$	704.17	704.17	1.32
Error	5	$[\psi'] - [T']$	2672.83	534.57	--

Note, again, the F ratio is correct, but to get back to the original sums of squares we need to normalize by dividing by $n \cdot \sum c_j^2$.

$$SS_{A_{\text{comp}}} = 704.17/6 = 117.36$$

$$SS_{SxA_{\text{comp}}} = 2672.83/6 = 445.47$$

C. Since A_{comp} and $A_{\text{comp}'}$ are a complete partition of A (and SxA_{comp} and $SxA_{\text{comp}'}$ are a complete partition of SxA) we can check that the sum of the SS's is what it should be:

$$SS_A = 30.09 + 117.36 = 147.45$$

$$SS_{SxA} = 354.42 + 445.47 = 799.89$$

II. Two-way (and higher-way) contrasts on repeated measures.

Consider Lopes' (1976, *JEP: General*) poker experiment. The subject rates (scale of 1-10) the probability of beating a pair of opposing stimulus hands (Hand 1 x Hand 2 design) with a pair of sevens. Look at $H1_{\text{LINEAR}} \times H2_{\text{LINEAR}}$.

		Hand 2							
		Strong		Weak					
Hand 1	Strong	P ₁	1	P ₁	1	P ₁	2	3	Data Table
		P ₂	2	P ₂	2	P ₂	3	3	
		P ₃	1	P ₃	2	P ₃	2	3	
		P ₄	1	P ₄	1	P ₄	2	2	
		P ₅	2	P ₅	2	P ₅	3	3	
	Weak	P ₁	1		2		4	7	
		P ₂	2		2		5	6	
		P ₃	2		3		4	6	
		P ₄	1		2		3	6	
		P ₅	2		3		5	7	
P ₁	1		3		7	10			
P ₂	2		4		6	9			
P ₃	1		4		7	9			
P ₄	2		3		8	10			
P ₅	2		3		7	9			

		X				
		-3	-1	1	3	
-1		3	1	-1	-3	Table of Coefficients of L x L
0		0	0	0	0	
1		-3	-1	1	3	

1. Make a table of $\hat{\psi}$ s:

S1	24	2. Compute $[\psi] = \sum \psi^2 / 1 = 2444$
S2	19	
S3	21	3. Compute $[T] = 110^2 / 5 = 2420$
S4	25	
S5	<u>21</u>	4. Compute $[\psi] - [T] = 24$

$$\sum \psi = 110$$

5. Source	df	EQ	SS	MS	F
Mean	1	[T]	2420	2420	403.33
Error	4	$[\psi] - [T]$	24	6	

If you want you may normalize by dividing the SSs by $n \cdot \sum c_j^2 = 40$.

III. Partial Interactions for Repeated Measures

Note – A new set of data begins here.

	A1	A2	A3	A4
B1	1	3	7	10
	2	4	6	9
	1	4	7	9
	1	3	8	10
	2	3	7	9
B2	2	7	8	3
	4	4	7	4
	4	4	5	5
	3	6	4	4
	2	3	6	6

Suppose we wish to test the hypothesis that the linear effect for factor A differs at the two levels of B (i.e., to test the $A_{\text{LINEAR}} \times B$ partial interaction).

	C1 $\psi@B1$	C2 $\psi@B2$	Because there are only 2 levels B, this is same as $A_{\text{LINEAR}} \times B_{\text{LINEAR}}$.
S1	31	4	Coefficients are: -3, -1, 1, 3
S2	23	3	
S3	27	4	Notice that we have now a simple one-way design with repeated measures. If there is a significant partial interaction then there will be a significant difference between the two columns when tested against the S x Column interaction.
S4	32	1	
S5	<u>25</u>	<u>15</u>	
	138	27	

Because this is a one df contrast, it can be done as a one group anova by taking C1 – C2.

1. Compute the basic ratios

$$\begin{aligned}
 [T] &= 165^2/10 = 2722.5 \\
 [SC] &= 4135/1 = 4135.0 \\
 [C] &= 19773/5 = 3954.6
 \end{aligned}$$

$$[S] = 5551 / 2 = 2775.5$$

$$2. \quad SS_C = 3954.6 - 2722.5 - 1232.1$$

$$SS_{SxC} = 4135.0 - 3954.6 - 2775.5 + 2722.5 = 127.4$$

$$3. \quad MS_C = 1232.1 / 1 = 1232.1$$

$$MS_{SxC} = 127.4 / 4 = 31.85$$

$$4. \quad F_C = F_{A|INxB} = 1232.1 / 31.85 = 38.68$$

To normalize, divide by $n * \sum c_j^2 = 20$.

V. Simple Main Effects and Simple Interaction Effects

Simply divide the data set according to the factor or factors being ignored, and run the regular repeated measures ANOVA. This will give the correct analysis including error terms.

For example, to compute the F ratio for A at B₁, simply separate out the data for B₁ and test factor A as though it were a one-way design.

Poker Example

H₂ @ H₁ Strong

Source	df	SS
Mean	1	84.05
Subjects	4	3.20
A	3	6.55
A x S	12	1.20

Partitions:

$$A = A + AxB$$

$$AxS = AxS + AxBxS$$

Mean: Mean + B

S: S + BxS

H₂ @ H₁ Medium

Source	df	SS
Mean	1	266.45
Subjects	4	3.30
A	3	68.15
A x S	12	3.10

H₂ @ H₁ Weak

Source	df	SS
Mean	1	572.45
Subjects	4	.80
A	3	184.95
A x S	12	4.8