

Partitions as Sets of Contrasts

A standard anova of a two-way design is really a partition that can be expressed as a set of orthogonal contrasts. Subsets of the SS for the contrasts are summed to form the SS for the two main effects and interaction in score in 3 x 3 design.

	A ₁			A ₂			A ₃			Contrast #	
	B ₁	B ₂	B ₃	B ₁	B ₂	B ₃	B ₁	B ₂	B ₃		
Main	2	2	2	-1	-1	-1	-1	-1	-1	1	Standard Partition
Effect A	0	0	0	1	1	1	-1	-1	-1	2	
Main	2	-1	-1	2	-1	-1	2	-1	-1	3	
Effect B	0	1	-1	0	1	-1	0	1	-1	4	
A x B	4	-2	-2	-2	1	1	-2	1	1	5	
	0	0	0	2	-1	-1	-2	1	1	6	
	0	2	-2	0	-1	1	0	-1	1	7	
	0	0	0	0	1	-1	0	-1	1	8	
B @ A ₁	2	-1	-1	0	0	0	0	0	0	1'	
	0	1	-1	0	0	0	0	0	0	2'	
B @ A ₂	0	0	0	2	-1	-1	0	0	0	3'	
	0	0	0	0	1	-1	0	0	0	4'	
B @ A ₃	0	0	0	0	0	0	2	-1	-1	5'	
	0	0	0	0	0	0	0	1	-1	6'	

The simple main effects of B @ each level of A are given as contrasts in the bottom portion of the table. Because there are 6 contrasts, they use 6 df.

Notice that the contrasts in the top can be expressed as linear functions of the contrasts in the bottom, and vice versa.

Example:

$$\begin{aligned} \#3 &= 1' + 3' + 5' \\ \#5 &= 2(1') + (-1)(3') + (-1)(5') \\ \#1' &= (\#3) + (-1)(\#6) \\ \#6' &= _[(2)(\#4) - (\#7) - (\#8)] \end{aligned}$$

Main Effect A contrasts are orthogonal to #s 1'-6'.