<u>Derivation of coefficients for Trend Analysis</u>

(1) $\mu_i = \beta_1(a_1 + X_j) + \beta_2(a_2 + b_2X_j + X_j^2) + \beta_3(a_3 + b_3X_j + c_3X_j^2 + X_j^3)$ + etc.

-- Assume an experiment with 4 age groups – 8, 9, 10, and 11-year-olds. We want to do trend analysis on Age. So the ages are used as the values of X.

For linear: $c_j = a_1 + X_j$ (from Eq. 1). Plug in the age as X_j:

	$c_1 = a_1 + 8$	$c_1 = -9.5 + 8 = -1.5$
Add	$c_2 = a_1 + 9$	$c_2 = -9.5 + 9 = -.5$
these	$c_3 = a_1 + 10$	$c_3 = -9.5 + 10 = .5$
	$c_4 = a_1 + 11$	$c_4 = -9.5 + 11 = 1.5$

$\sum c_j = 4a_1 + 38$

$0 = 4a_1 + 38$; because $\sum c_j = 0$

$-38 = 4a_1$

$a_1 = -9.5$ Now plug in –9.5 for a_1

-- So, coefficients for linear are –1.5, -.5, .5, 1.5. If we multiply by 2, we get the values from Keppel Table A –4, -3, -1, 1, 3.

-- Next we want to find the coefficients for quadratic trend. We want these coefficients to be orthogonal to the linear coefficients.

For quadratic: $c_j = a_2 + b_2X_j + X_j^2$ (from Eq. 1). Plug in the age as X_j:

	$c_1 = a_2 + (b_2)(8) + 8^2$	$c_1 = 89 + (-19)(8) + 64 = 1$
Add	$c_2 = a_2 + (b_2)(9) + 9^2$	$c_2 = 89 + (-19)(9) + 81 = -1$
these	$c_3 = a_2 + (b_2)(10) + 10^2$	$c_3 = 89 + (-19)(10) + 100 = -1$
	$c_4 = a_2 + (b_2)(11) + 11^2$	$c_4 = 89 + (-19)(11) + 121 = 1$

$\sum c_j = 4a_2 + 38b_2 + 366$

Now apply orthogonality constraint. $\sum c_j c_j' = 0$.

So we multiply linear c_j by the quad c_j :

	(linear c_j)	(quad c_j)
	(-3)	$(a_2 + 8b_2 + 64)$
Add	(-1)	$(a_2 + 9b_2 + 81)$
these	(1)	$(a_2 + 10b_2 + 100)$
	(3)	$(a_2 + 11b_2 + 121)$

$\sum c_j c_j' = 0a_2 + 10b_2 + 190$

-- Now we have 2 equations in 2 unknowns:

(2) $0 = 4a_2 + 38b_2 + 366$

$0 = 0a_2 + 10b_2 + 190$; or, $b_2 = -19$

Put $b_2 = -19$ into Eq. 2: $0 = 4a_2 + (38)(-19) + 366$

$a_2 = 89$

-- Now plug $a_2 = 89$ and $b_2 = -19$ into quadratic expressions above and calculate c_j for quad. This yields the values in Keppel's table.

-- This method works for <u>unevenly</u> spaced independent variables but will yield different c_j from those in the table in Keppel, of course.