Psych 610 Handout #19, p. 1
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NOTES ON TWO-WAY TREND ANALYSIS

I. Here's a way to see that the coefficients for a two-way linear x linear trend are a filter for the linear x linear portion of an interaction.

Below is a set of means. Note that each mean is simply the value of the row times the value of the column.

	1	2	3	4	Y_R
3	3	6	9	12	7.5
2	2	4	6	8	5.0
1	1	2	3	4	2.5
Y_C	2	4	6	8	

This would plot as a fan of diverging straight lines (see Fig. 1).

NOTE that there are main effects for the column factor. Since we are interested in the interaction and not the main effects, we can get rid of them by subtracting Y_{Cj} from each Y_{RCjk}

	1	2	3	4	Y_R
3	1	2	3	4	2.5
2	0	0	0	0	0
1	-1	-2	-3	-4	-2.5
Y_C	0	0	0	0	

This would plot as shown in Fig. 2.

Now we have no main effects for column, but we still have main effects for row. So, let's subtract Y_{Rk} from each Y_{RCjk}. This gives:

-1.5	-.5	.5	1.5	0
0	0	0	0	0
1.5	.5	-.5	-1.5	0
0	0	0	0	

This would plot as shown in Fig. 3. Note that this is exactly the same shape as the set of coefficients for a linear x linear trend analysis.

II. Here's a way to see that version of the denominator taught in class for a two-way trend test gives you the same value as Keppel's way.

<u>Class version</u>	<u>Keppel's version</u>
$SS_{TxT} = n(\sum c_{jk} Y_{AjBk})^2 / \sum c_{jk}^2$	$SS_{TxT} = n(\sum c_{jk} Y_{AjBk})^2 / (\sum c_j^2)(\ \sum c_k^2)$

$c_{jk} = c_j \times c_k$

	-3	-1	1	3	
-1	3	1	-1	-3	$\sum c_{jk}^2 = 40$
0	0	0	0	0	$(\sum c_j^2)(\sum c_k)^2 = 40$
1	-3	-1	1	3	

<u>Class Way</u>:

$\sum c_{jk}^2 = \sum (c_j c_k)^2$

Assume that c_j has two levels, c_x, c_y; and c_k has three levels, c_a, c_b, c_d.

We would have

$\sum c_{jk}^2 = c_x^2 c_a^2 + c_x^2 c_b^2 + c_x^2 c_d^2 + c_y^2 c_z^2 + c_y^2 c_b^2 + c_y^2 c_d^2$

This can be rearranged as follows:

$$\begin{aligned}\sum c_{jk}^2 &= c_x^2(c_a^2 + c_b^2 + c_d^2) + c_y^2(c_a^2 + c_b^2 + c_d^2) \\ &= (c_x^2 + c_y^2)(c_a^2 + c_b^2 + c_d^2) \\ &= (\sum c_j^2)(\sum c_k^2)\end{aligned}$$

But this is just Keppel's way.

So the method taught in class and Keppel's way are the same in terms of the answer they give.