Unequal N

A 2 x 2 example to illustrate nonorthogonality of unequal n designs

	A_1		A_2		
	B_1	B_2	B_1	B_2	Contrast
Main A	1	1	-1	-1	X_1
Main B	1	-1	1	-1	X_2
A x B	1	-1	-1	1	X_3

Assume unequal (and unbalanced) n

Total n = 12

		A	
		A1	A2
B	B1	4	3
	B2	3	2

cell ns

Regular anova model: $Y_{ijk} = \mu + \alpha_j + \beta_k + \alpha\beta_{jk} + \varepsilon_{ijk}$.

X_1 is the predictor variable which replaces α_j in multiple regression. For Ss in cell A_1B_1, X_1 has value 1. For Ss in cell A_2B_1, X_1 = -1, etc.

X_2 is the predictor variable for B_k.

X_3 is the predictor variable for $\alpha\beta_{jk}$.

Mult. R model:

$$Y_{ij} = a + \beta_1X_1 + \beta_2X_2 + \beta_3X_3 + \varepsilon.$$

The estimate of intecept, a, will be the grand mean

		Main A	Main B	A x B
		X_1	X_2	X_3
A_1B_1	S_1	1	1	1
	S_2	1	1	1
	S_3	1	1	1
	S_4	1	1	1
A_1B_2	S_5	1	-1	-1
	S_6	1	-1	-1
	S_7	1	-1	-1
A_2B_1	S_8	-1	1	-1
	S_9	-1	1	-1
	S_{10}	-1	1	-1
A_2B_2	S_{11}	-1	-1	1
	S_{12}	-1	-1	1

Are predictor variables X_1, X_2, and X_3 orthogonal?

Find $\sum c_j c_k$.

$$\sum X_1X_2 = (4 + -3 + -3 + 2) = 0$$

$$\sum X_1X_3 = (4 + -3 + 3 - 2) = 2$$

$$\sum X_2X_3 = (4 + 3 - 3 - 2) = 2$$

X_1 and X_2 are orthogonal, but X_1 and X_3 are not, nor are X_2 and X_3.

Equal n: imagine 4 Ss/cell

$$\sum X_1X_2 = (4 - 4 - 4 + 4) = 0$$

$$\sum X_1X_3 = (4 - 4 + 4 - 4) = 0$$

$$\sum X_2X_3 = (4 + 4 - 4 - 4) = 0$$