

Analysis of variance for Unbalanced Between Groups designs in R

For Psychology 610 University of Wisconsin--Madison

R is very touchy about unbalanced designs, partly because it includes several ways of calculating the SS. The Type I method is the default in the 'aov' module. Type II and Type III sums of squares can be tested using 'Anova' (note capital A) in the 'car' package. Install the 'car' package in R, then activate it for your session using the 'library(car)' command.

When using R for unbalanced designs, *the contrast coding of your factors is absolutely critical*. I illustrate how dramatically this can alter your results below. Other programs are more idiot-proof for unbalanced designs.

Contents of this tutorial:

- I. Bring in data and arrange for analysis. This section includes some data transformation tricks for recoding variables, and constructing centered dummy codes for main effects and interactions.
- II. Anova with Type I, Type II and Type III SS's.
- III. Same analyses but conducted with multiple regression using the codes laboriously constructed in Part I.
- IV. Non-orthogonality of the design illustrated by correlating the main effect contrast codes across all observations. Shows how to construct a correlation matrix.

Summary of R steps:

Step 1) set options for contrasts to 'sum' or 'poly',

Step 2) run anova in 'aov',

Step 3) use 'Anova' (capital A) in the 'car' package to extract Type II or Type III SS's from the 'aov' run.

Summary of R code:

```
> options(contrasts=c("contr.sum","contr.poly")) # set the contrast type R will use as a default
```

```
> model = aov(dv ~ A*B*C) # make sure A, B and C are 'factors'
```

```
> library(car) # activate the 'car' package
```

```
> Anova(model,type=c("III"))
```

Warnings: 1) Type III SS's are sensitive to the specific contrasts used. 2) make sure you make your independent variables into factors.

An Example in detail

An Unbalanced Between Groups anova is illustrated here with data from a 2(sex of animal) x 2(prenatal alcohol exposed or not) x 2(prenatal stress exposed or not). The dependent variable is fallypride uptake in the striatum, a measure of dopamine system function in that area of the brain.

I. Bring in data and set up the grouping variable codes. The original data set had sex coded as 1 and 2, and the animal's condition was coded 1=alc only, 2=control, 3=stress only, and 5=alc+stress. These codes needed to be turned into two separate variables for alcohol and stress. I did this in a very plodding manner to make sure I didn't make mistakes. Notice that I have entered 'NaN' for missing data.

```
> pet1.data=read.table(pipe("pbpaste"),header=T) # I pasted the data to the clipboard
```

```
> pet1.data
```

	ID	sex	condition	fal	fmt
1	AR54	2	1	12.88000	6.32000
2	AR56	2	1	14.80000	5.38000
3	AR58	1	1	11.46000	6.64000
4	AR61	1	2	13.48800	6.25700
5	AR66	2	3	17.34000	4.73000
6	AR67	1	3	12.05000	5.78000
7	AR69	1	2	10.15000	4.93600
8	AR70	2	5	18.94000	7.22000
9	AR71	2	3	16.67000	4.74000
10	AR72	1	5	14.39000	5.55000
11	AR76	2	2	14.85000	7.55000
12	AR77	2	2	9.37000	5.59000
13	AR78	2	3	11.91330	5.20512
14	AR80	1	2	14.61000	8.34000
15	AR82	1	1	12.41000	6.76000
16	AR87	1	1	13.92000	5.63000
17	AR88	2	3	17.10000	7.39000
18	AR89	1	2	11.35000	5.91000
19	AR95	2	2	10.53000	5.78000
21	AS03	1	2	NaN	NaN
22	AS06	2	5	23.07000	5.90000
24	AS47	1	1	14.49000	4.96000
25	AS48	1	5	19.26000	5.26000
26	AS51	1	2	15.72000	5.89500
29	AS59	1	2	18.73730	4.28064
30	AS61	1	1	12.85910	4.38298
31	AS62	1	2	11.79000	6.07000
32	AS64	1	1	13.72260	4.79091
34	AS80	2	1	8.98263	5.33459
36	AS89	1	2	14.53000	6.85000
37	AT35	1	5	14.69000	5.25000
40	AT83	1	3	17.04100	3.76925
41	AT85	2	3	19.03000	5.36000
42	AT86	1	1	12.04270	4.39087
43	AT94	2	5	11.70000	6.51000
44	AU03	2	5	18.74000	6.77000
45	AU08	2	3	14.09000	4.89000
46	AU09	2	5	9.67000	5.91000
47	AU11	1	5	15.03000	4.63000

A. Create codes for the prenatal stress variable, -1 and 1 for no stress and stress exposed. I call it 'tress' so it won't have the first letter 's', and I won't confuse it with sex of animal.

> `pet2.data=transform(pet1.data, tress=ifelse(condition > 2, 1,-1))` # the ifelse statement sets the value of the named variable for a whole vector according whether the test in parentheses (`condition > 2`) is true or false.

> `pet2.data` # I want to see the results before I continue

```
ID sex condition      fal      fmt      tress
1  AR54  2          1 12.88000  6.32000     -1
2  AR56  2          1 14.80000  5.38000     -1
3  AR58  1          1 11.46000  6.64000     -1
4  AR61  1          2 13.48800  6.25700     -1
5  AR66  2          3 17.34000  4.73000      1
6  AR67  1          3 12.05000  5.78000      1
. . .
44 AU03  2          5 18.74000  6.77000      1
45 AU08  2          3 14.09000  4.89000      1
46 AU09  2          5  9.67000  5.91000      1
47 AU11  1          5 15.03000  4.63000      1
```

B. Create codes for the prenatal alcohol variable, -1 and 1 for no alc and alc exposed. This is trickier because the original conditions aren't in order. I folded the original condition in the middle by subtracting a constant and then taking the absolute value. This is a good trick to learn for recoding variables.

> `pet3.data=transform(pet2.data,alc=ifelse(abs(condition - 3) >= 2,1,-1))`

> `pet3.data`

```
ID sex condition      fal      fmt tress alc
1  AR54  2          1 12.88000  6.32000     -1  1
2  AR56  2          1 14.80000  5.38000     -1  1
3  AR58  1          1 11.46000  6.64000     -1  1
4  AR61  1          2 13.48800  6.25700     -1 -1
5  AR66  2          3 17.34000  4.73000      1 -1
6  AR67  1          3 12.05000  5.78000      1 -1
. . .
44 AU03  2          5 18.74000  6.77000      1  1
45 AU08  2          3 14.09000  4.89000      1 -1
46 AU09  2          5  9.67000  5.91000      1  1
47 AU11  1          5 15.03000  4.63000      1  1
```

C. Create codes for sex of animal, -1 for females, 1 for males.

> `pet4.data=transform(pet3.data,sexcode=ifelse(sex==1,-1,1))`

> `pet4.data`

```
ID sex condition      fal      fmt tress alc sexcode
1  AR54  2          1 12.88000  6.32000     -1  1      1
2  AR56  2          1 14.80000  5.38000     -1  1      1
3  AR58  1          1 11.46000  6.64000     -1  1     -1
4  AR61  1          2 13.48800  6.25700     -1 -1     -1
. . .
45 AU08  2          3 14.09000  4.89000      1 -1      1
46 AU09  2          5  9.67000  5.91000      1  1      1
47 AU11  1          5 15.03000  4.63000      1  1     -1
```

D. Create interaction codes. You don't need to create the next codes, which represent all the interactions by multiplying codes together. I am doing this solely for instructional purposes, so we can see how to calculate Type III SS's using multiple regression later.

```
> attach(pet4.data) # attach the data set to simplify naming variables for calculations
> alcXtress=alc*tress
> alcXsex=alc*sexcode
> tressXsex=tress*sexcode
> way3=alc*tressXsex
```

E. Now combine the interaction codes with the data:

```
> pet5.data=cbind(pet4.data,alcXtress,alcXsex,tressXsex,way3) # cbind means "column
bind" variables together
> pet5.data
```

	ID	sex	condition	fal	fmt	tress	alc	sexcode	alcXtress	alcXsex	tressXsex	way3
1	AR54	2	1	12.88000	6.32000	-1	1	1	-1	1	-1	-1
2	AR56	2	1	14.80000	5.38000	-1	1	1	-1	1	-1	-1
3	AR58	1	1	11.46000	6.64000	-1	1	-1	-1	-1	1	1
4	AR61	1	2	13.48800	6.25700	-1	-1	-1	1	1	1	-1
5	AR66	2	3	17.34000	4.73000	1	-1	1	-1	-1	1	-1
. . .												
43	AT94	2	5	11.70000	6.51000	1	1	1	1	1	1	1
44	AU03	2	5	18.74000	6.77000	1	1	1	1	1	1	1
45	AU08	2	3	14.09000	4.89000	1	-1	1	-1	-1	1	-1
46	AU09	2	5	9.67000	5.91000	1	1	1	1	1	1	1
47	AU11	1	5	15.03000	4.63000	1	1	-1	1	-1	-1	-1

Get rid of extraneous things, and then attach the final data set.

```
> rm(pet4.data)
> detach(pet4.data)
> attach(pet5.data)
```

The following object(s) are masked `_by_` .GlobalEnv :

```
alcXsex alcXtress tressXsex way3
```

F. Save the constructed data set in your computer so you never have to go through this re-coding again.

```
> library(MASS) # activate the 'MASS' package
> write.matrix(pet5.data,file="petwithcodes",sep=" ") # I like a blank as a separator
because it can be read by some legacy statistical software I use, but many prefer a comma
because files that are comma-delimited can be read by many many programs very easily.
```

II. Create factors and carry out anova using 'aov'

```
> Alc=factor(alc)
> Tress=factor(tress)
> Sex=factor(sexcode)
```

A. Run ‘aov’, and look at the default output from ‘aov’. This gives *Type I SS*. In Type I SS, the order of entry of the factors matters, and almost no one is interested in these. Here I show how order affects the results in this example.

> `modfull=aov(fal~Alc*Tress*Sex)` # this specifies the full model with all possible main effects and interactions of the 3 factors.

> `summary(modfull,intercept=T)` # ask for the summary. This gives **Type I** Sums of Squares, not the Type III we are used to in Psychology.

```

      Df Sum Sq Mean Sq  F value    Pr(>F)
(Intercept)  1  7771.1   7771.1  834.9437 < 2.2e-16 ***
Alc           1    0.2     0.2    0.0206  0.886908
Tress        1   81.0    81.0    8.7021  0.006111 **
Sex          1    1.4     1.4    0.1547  0.696840
Alc:Tress    1    1.7     1.7    0.1863  0.669088
Alc:Sex      1    0.8     0.8    0.0865  0.770642
Tress:Sex    1   11.4    11.4    1.2242  0.277330
Alc:Tress:Sex 1    2.7     2.7    0.2880  0.595437
Residuals   30   279.2     9.3
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
1 observation deleted due to missingness
    
```

Type I SS’s are sensitive to the order in which the factors are entered. Here I reordered the factors in the ‘aov’ statement. The SS below are close, but notice that Alc is reduced in size, and Tress is a touch larger.

> `modreorder=aov(fal~Tress*Alc*Sex)`

> `summary(modreorder,intercept=T)`

```

      Df  Sum Sq Mean Sq  F value    Pr(>F)
(Intercept)  1  7771.1   7771.1  834.9437 < 2.2e-16 ***
Tress        1   81.2    81.2    8.7225  0.006058 **
Alc          1  0.001548  0.001548  0.0002  0.989796
Sex          1    1.4     1.4    0.1547  0.696840
Tress:Alc    1    1.7     1.7    0.1863  0.669088
Tress:Sex    1   11.9    11.9    1.2803  0.266807
Alc:Sex      1    0.3     0.3    0.0305  0.862640
Tress:Alc:Sex 1    2.7     2.7    0.2880  0.595437
Residuals   30   279.2     9.3
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
1 observation deleted due to missingness
    
```

B. Means and standard errors.

> `model.tables(modfull,"means",se=T)` # R tells us that the design is unbalanced, and so we need to use another method to get the standard errors.

```

Design is unbalanced - use se.contrast() for se's
Tables of means
Grand mean
14.30044

Alc
      -1      1
      14.23 14.37
rep 19.00 19.00
    
```

R gives *weighted means*, that is the marginals are weighted by the cell n.

To calculate *unweighted means*, you just average together the cell means that go into the marginal you want (i.e., take the mean of the means). There might be a function to calculate unweighted means in R, but I haven’t found it yet.

```
Tress
  -1    1
 12.99 15.92
rep 21.00 17.00

Sex
  -1    1
 14.46 14.1
rep 21.00 17.0

Alc:Tress
  Tress
Alc  -1    1
  -1 13.173 15.682
  rep 11.000 8.000
  1 12.779 16.141
  rep 10.000 9.000

Alc:Sex
  Sex
Alc  -1    1
  -1 14.522 13.905
  rep 10.000 9.000
  1 14.417 14.309
  rep 11.000 8.000

Tress:Sex
  Sex
Tress -1    1
  -1 13.427 11.894
  rep 15.000 6.000
  1 15.379 16.217
  rep 6.000 11.000

Alc:Tress:Sex
, , Sex = -1

  Tress
Alc  -1    1
  -1 13.797 14.546
  rep 8.000 2.000
  1 12.986 15.843
  rep 7.000 4.000

, , Sex = 1

  Tress
Alc  -1    1
  -1 11.583 16.024
  rep 3.000 6.000
  1 12.221 16.424
  rep 3.000 5.000
```

C. Use ‘Anova’ (with capital ‘A’) in ‘car’ package to get Type II and/or Type III SS
. This is where things are tricky. Type III SS solutions depend on the contrasts that R uses internally. We used the factors, not our contrast codes, in the ‘aov’ run. R made its own default contrast coding. The best thing to do is to set the options in R:

```
> options(contrasts=c("contr.sum", "contr.poly")) # you MUST specify this before you
run 'aov'
> modfull=aov(fal~ Alc*Tress*Sex)
> library(car) # activate the 'car' package
> Anova(modfull,type=c("III")) # ask for type III SS from the aov run we did just above.
```

Anova Table (Type III tests)

```
Response: fal
      Sum Sq Df F value    Pr(>F)
(Intercept) 6271.9 1 673.8665 < 2.2e-16 ***
Alc           1.1 1  0.1217  0.729664
Tress        73.1 1  7.8583  0.008783 **
Sex           0.4 1  0.0443  0.834804
Alc:Tress     1.7 1  0.1832  0.671697
Alc:Sex       0.1 1  0.0159  0.900448
Tress:Sex    12.4 1  1.3300  0.257915
Alc:Tress:Sex 2.7 1  0.2880  0.595437
Residuals    279.2 30
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

***** If you run the 'aov' analysis without specifying the contrasts 'options' given above, you'll get pretty strange results for Type III SS's. Here's an example of what I got without the options statement set properly. Whoops, where'd my significant effect of Tress go? And all the SS's are quite different, except the residual and the 3-way interaction, which do match.**

Incorrect analysis shown here for illustration:

Anova Table (Type III tests)

```
Response: fal
      Sum Sq Df F value    Pr(>F)
(Intercept) 1522.84 1 163.6171 1.113e-13 ***
Alc           2.45 1  0.2635  0.6115
Tress         0.90 1  0.0963  0.7584
Sex          10.69 1  1.1486  0.2924
Alc:Tress     4.36 1  0.4689  0.4988
Alc:Sex       2.24 1  0.2411  0.6270
Tress:Sex    12.12 1  1.3018  0.2629
Alc:Tress:Sex 2.68 1  0.2880  0.5954
Residuals    279.22 30
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Type II SS's are not as sensitive to the contrasts as Type III:

```
> Anova(modfull) # this will give type II SS tests, the default for 'Anova' (with a capital
A). Now the significant Tress effect is back, but which anova table is "correct"? The SS
don't match up, though they are in the ballpark. Actually, this issue is 'which do you
prefer to answer your research question'.
```

Anova Table (Type II tests)

```
Response: fal
      Sum Sq Df F value    Pr(>F)
```

```
Alc          0.061  1  0.0065 0.936201
Tress       79.366  1  8.5272 0.006582 **
Sex         1.071  1  0.1150 0.736854
Alc:Tress   1.738  1  0.1867 0.668775
Alc:Sex     0.283  1  0.0305 0.862640
Tress:Sex   11.394  1  1.2242 0.277330
Alc:Tress:Sex 2.681  1  0.2880 0.595437
Residuals  279.220 30
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

D. Re-do the anova in ‘aov’ but using our codes instead of having R create factors
> modfull2=aov(fal~alc+tress+sexcode+alcXtress+alcXsex+tressXsex+way3) # we list all the variables that we want in the analysis. Remember that we created the interaction codes and named them with the ‘X’ as part of the name earlier. The ‘X’ isn’t an operator. Also, these codes are contrast codes, so we use them directly without making factors out of them.

> summary(modfull2,intercept=T) # ask for the results, these will be Type I SS. These results match the first results exactly. ‘aov’ gives the same results (Type I SS) whether you use contrast codes you created yourself, or make factors and let R use those. Order of entry still matters for Type I SS.

```
          Df Sum Sq Mean Sq F value    Pr(>F)
(Intercept) 1 7771.1  7771.1 834.9437 < 2.2e-16 ***
alc          1    0.2    0.2  0.0206  0.886908
tress       1  81.0    81.0  8.7021  0.006111 **
sexcode     1   1.4    1.4  0.1547  0.696840
alcXtress   1   1.7    1.7  0.1863  0.669088
alcXsex     1   0.8    0.8  0.0865  0.770642
tressXsex   1  11.4   11.4  1.2242  0.277330
way3       1   2.7    2.7  0.2880  0.595437
Residuals  30 279.2    9.3
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
1 observation deleted due to missingness
```

Next, use ‘Anova’ to get Type II and Type III SS solutions from the run using the our own coded variables. The Type III SS are now in the same ballpark as the Type I and Type II SS. Not only that, but the Type III SS here will match those from SPSS, SAS, or my legacy DOS software, BMDP. But now the Type II SS don’t match the Type II SS from before; Type II and Type III SS match each other now.

Main point: the coding of your predictor variables is *critical* when you ask for Type III SS in R !!! But we still face the question of which is better for our purposes.

> Anova(modfull2,type=c("III"))

```
Anova Table (Type III tests)

Response: fal
          Sum Sq Df F value    Pr(>F)
(Intercept) 6271.9  1 673.8665 < 2.2e-16 ***
alc          1.1  1   0.1217  0.729664
```



```
tress      73.1  1   7.8583  0.008783 **
sexcode    0.4  1   0.0443  0.834804
alcXtress  1.7  1   0.1832  0.671697
alcXsex    0.1  1   0.0159  0.900448
tressXsex  12.4  1   1.3300  0.257915
way3       2.7  1   0.2880  0.595437
Residuals 279.2 30
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

> Anova(modfull2)

Anova Table (Type II tests)

```
Response: fal
      Sum Sq Df F value    Pr(>F)
alc      1.132  1  0.1217  0.729664
tress   73.140  1  7.8583  0.008783 **
sexcode  0.412  1  0.0443  0.834804
alcXtress 1.705  1  0.1832  0.671697
alcXsex  0.148  1  0.0159  0.900448
tressXsex 12.379  1  1.3300  0.257915
way3     2.681  1  0.2880  0.595437
Residuals 279.220 30
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

III. Re-do all the analyses using multiple regression ‘lm’ instead of ‘aov’. Type III SS find the variance accounted for by each variable *after* the other variables are entered in the model. With all these interactions, this will be a nuisance, and we’ll be grateful for the Type III option in the ‘car’ package.

A. First run the full model in ‘lm’, and get the R-squared of the regression run. You can also get the analysis of variance table of the regression by using ‘anova’ (with a small ‘a’).

> full=lm(fal~alc+tress+sexcode+alcXtress+alcXsex+tressXsex+way3)
 > summary(full)

```
Call:
lm(formula = fal ~ alc + tress + sexcode + alcXtress + alcXsex +
    tressXsex + way3)
```

```
Residuals:
      Min       1Q   Median       3Q      Max
-6.7540 -1.8320  0.2594  1.8182  6.6460
```

```
Coefficients:
      Estimate Std. Error t value Pr(>|t|)
(Intercept)  14.1779     0.5462  25.959 < 2e-16 ***
alc           0.1905     0.5462   0.349  0.72966
tress        1.5311     0.5462   2.803  0.00878 **
sexcode     -0.1149     0.5462  -0.210  0.83480
alcXtress    0.2338     0.5462   0.428  0.67170
alcXsex      0.0689     0.5462   0.126  0.90045
```

```
tressXsex    0.6299    0.5462    1.153    0.25791
way3         -0.2931    0.5462   -0.537    0.59544
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3.051 on 30 degrees of freedom
(1 observation deleted due to missingness)
Multiple R-squared: 0.2622,    Adjusted R-squared: 0.09007
F-statistic: 1.523 on 7 and 30 DF,  p-value: 0.1974
```

B. Now run the regression again, but omitting in turn each main effect term.

```
> alcmain=lm(fal~tress+sexcode+alcXtress+alcXsex+tressXsex+way3)
> summary(alcmain)
```

```
Call:
lm(formula = fal ~ tress + sexcode + alcXtress + alcXsex + tressXsex +
    way3)
```

```
Residuals:
    Min       1Q   Median       3Q      Max
-6.6054 -1.8983  0.2506  1.7751  6.7946
```

```
Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  14.19638    0.53584   26.494 < 2e-16 ***
tress         1.55283    0.53484    2.903  0.00675 **
sexcode      -0.13955    0.53385   -0.261  0.79551
alcXtress    0.21685    0.53625    0.404  0.68871
alcXsex       0.06747    0.53836    0.125  0.90108
tressXsex    0.60189    0.53254    1.130  0.26705
way3        -0.22048    0.49770   -0.443  0.66084
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
Residual standard error: 3.007 on 31 degrees of freedom
(1 observation deleted due to missingness)
Multiple R-squared: 0.2592,    Adjusted R-squared: 0.1159
F-statistic: 1.808 on 6 and 31 DF,  p-value: 0.1300
```

```
> tressmain=lm(fal~alc+sexcode+alcXtress+alcXsex+tressXsex+way3) # omit 'tress'
> summary(tressmain)
```

```
Call:
lm(formula = fal ~ alc + sexcode + alcXtress + alcXsex + tressXsex +
    way3)
```

```
Residuals:
    Min       1Q   Median       3Q      Max
-5.5597 -2.2337  0.0535  1.5501  7.8403
```

```
Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  14.0420    0.6012   23.357 <2e-16 ***
alc           0.3656    0.5996    0.610  0.547
sexcode      0.4689    0.5580    0.840  0.407
alcXtress    0.3822    0.6007    0.636  0.529
alcXsex     -0.1559    0.5970   -0.261  0.796
tressXsex    0.6183    0.6035    1.024  0.314
way3       -0.4913    0.5985   -0.821  0.418
---

```

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3.371 on 31 degrees of freedom
 (1 observation deleted due to missingness)
 Multiple R-squared: 0.06896, Adjusted R-squared: -0.1112
 F-statistic: 0.3827 on 6 and 31 DF, p-value: 0.8844

> sexmain=lm(fal~alc+tress+alcXtress+alcXsex+tressXsex+way3) # omit sex
 > summary(sexmain)

Call:
 lm(formula = fal ~ alc + tress + alcXtress + alcXsex + tressXsex +
 way3)

Residuals:
 Min 1Q Median 3Q Max
 -6.8436 -1.8288 0.2233 1.8762 6.5564

Coefficients:
 Estimate Std. Error t value Pr(>|t|)
 (Intercept) 14.17879 0.53767 26.371 <2e-16 ***
 alc 0.20538 0.53316 0.385 0.7027
 tress 1.48724 0.49706 2.992 0.0054 **
 alcXtress 0.25064 0.53185 0.471 0.6408
 alcXsex 0.05777 0.53515 0.108 0.9147
 tressXsex 0.64007 0.53556 1.195 0.2411
 way3 -0.30626 0.53416 -0.573 0.5705

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3.003 on 31 degrees of freedom
 (1 observation deleted due to missingness)
 Multiple R-squared: 0.2611, Adjusted R-squared: 0.1181
 F-statistic: 1.826 on 6 and 31 DF, p-value: 0.1264

C. Subtract R-squared's by hand to get the contribution of each variable after all the other variables are included, **or use the 'anova' (small 'a')** function to have R calculate the significance of the R-squared difference for you. Notice that we only calculated the R-squared values that we need for the 3 main effects here. In order to calculate the interaction SS's you would have to omit each interaction term and obtain the R-sq value.

> anova(full,alcmain) # by listing two models, R will compare them. The SS difference is negative, but it doesn't matter. List them in the opposite order in the 'anova' statement
 Analysis of Variance Table

```
Model 1: fal ~ alc + tress + sexcode + alcXtress + alcXsex + tressXsex + way3
Model 2: fal ~ tress + sexcode + alcXtress + alcXsex + tressXsex + way3
  Res.Df  RSS Df Sum of Sq    F Pr(>F)
1     30 279.220
2     31 280.352 -1    -1.132 0.1217 0.7297
```

> anova(full,sexmain)
 Analysis of Variance Table

```
Model 1: fal ~ alc + tress + sexcode + alcXtress + alcXsex + tressXsex + way3
Model 2: fal ~ alc + tress + alcXtress + alcXsex + tressXsex + way3
```

```

    Res.Df    RSS Df Sum of Sq    F Pr(>F)
1      30 279.220
2      31 279.632 -1    -0.412 0.0443 0.8348
> anova(full,tressmain)
Analysis of Variance Table

Model 1: fal ~ alc + tress + sexcode + alcXtress + alcXsex + tressXsex +
  way3
Model 2: fal ~ alc + sexcode + alcXtress + alcXsex + tressXsex + way3
  Res.Df    RSS Df Sum of Sq    F    Pr(>F)
1      30 279.22
2      31 352.36 -1    -73.14 7.8583 0.008783 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
    
```

This laborious regression method creates Type III SS's and F's that match those from the Type III SS's that we calculated using our orthogonal contrast codes and 'Anova' (capital A).

IV. How non-orthogonal is this design?

There are two ways that I think about non-orthogonality.

First, do the SS add up the SS total? When a design is unbalanced, they $\sum SS > SS$ total. This is because some of the SS is overlapping. **Second**, do the predictor variables correlate with each other?

Second, we can look at the correlation between the predictor variables that we created. We constructed our codes for the main effects as -1 vs 1 contrasts, and all the interactions were constructed by multiplying the main effect contrast codes together. Two contrasts are not orthogonal is $\sum c_j * c_k \neq 0$. The cross product of two variables is the numerator of the correlation coefficient, so let's make a correlation table of the data.

The result shows that tress and sexcode are correlated 0.37. Also, tress and alc are correlated only .07. And alc correlates with the 'way3' the 3 way interaction codes. It would be nicer to have an orthogonal design wouldn't it?? Too bad we can't prenatally randomly assign animals to turn out to be male or female.

```
> cor(pet5.data, use="pairwise.complete.obs") # ask for correlation matrix
```

```

ID          sex  condition      fal      fmt      tress      alc
ID          NA      NA      NA      NA      NA      NA
sex         NA  1.00000000  0.25179072  0.110157454  0.188496160  0.37433155 -0.02917864
condition  NA  0.25179072  1.00000000  0.433514073  0.064119421  0.85445225  0.16953595
fal        NA  0.11015745  0.43351407  1.000000000  0.000745151  0.46315103  0.02249309
fmt        NA  0.18849616  0.06411942  0.000745151  1.000000000 -0.11583400 -0.04590049
tress      NA  0.37433155  0.85445225  0.463151029 -0.115834002  1.00000000  0.07427291
alc        NA -0.02917864  0.16953595  0.022493085 -0.045900495  0.07427291  1.00000000
sexcode    NA  1.00000000  0.25179072  0.110157454  0.188496160  0.37433155 -0.02917864
alcXtress  NA -0.11968254  0.47740149  0.076931948  0.333756000 -0.01595767 -0.12664952
alcXsex    NA -0.02917864  0.06402687  0.014046055  0.248399594 -0.13263019 -0.12894737
tressXsex  NA -0.08618128 -0.12598329  0.138699767  0.131278312 -0.08618128 -0.12824729
way3       NA  0.06149733 -0.03181471 -0.104682687  0.068917230 -0.04278075  0.38462756
ID          sexcode  alcXtress  alcXsex  tressXsex  way3
ID          NA      NA      NA      NA      NA
sex         1.00000000 -0.11968254 -0.02917864 -0.08618128  0.06149733
condition  0.25179072  0.47740149  0.06402687 -0.12598329 -0.03181471
fal        0.11015745  0.07693195  0.01404605  0.13869977 -0.10468269
fmt        0.18849616  0.33375600  0.24839959  0.13127831  0.06891723
tress      0.37433155 -0.01595767 -0.13263019 -0.08618128 -0.04278075
alc        -0.02917864 -0.12664952 -0.12894737 -0.12824729  0.38462756
    
```

```
sexcode    1.00000000 -0.11968254 -0.02917864 -0.08618128  0.06149733
alcXtress -0.11968254  1.00000000  0.38786415 -0.06000686 -0.11968254
alcXsex   -0.02917864  0.38786415  1.00000000  0.09404802 -0.13263019
tressXsex -0.08618128 -0.06000686  0.09404802  1.00000000  0.02585438
way3      0.06149733 -0.11968254 -0.13263019  0.02585438  1.00000000
Warning message:
In cor(pet5.data, use = "pairwise.complete.obs") :
  NAs introduced by coercion
```